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THE ALGEBRAIC ART OF DISCOURSE ALGEBRAIC *DISPOSITIO*, INVENTION AND IMITATION IN SIXTEENTH-CENTURY FRANCE

ABSTRACT

This paper is part of a research project devoted to inquiring into the connections between humanist rhetoric, dialectics and the teaching of the liberal arts on the one hand and the developments occurring in sixteenth-century algebra on the other. In this larger context, we have found that, especially in France, symbolic algebra as we know it grew out of mathematics within humanistic culture, and particularly out the interaction between mathematics and the disciplines of the text. This is what transformed algebra after its importation from Italy (and the German countries), so that it became what we call symbolic algebra.

The paper discusses first the way in which the disciplines of the text modified the way of writing algebra. Secondly, it looks at how one sixteenth-century author theorized mathematical creation in "literary" terms, as invention within imitation. To look at sixteenth-century algebra in this way necessitates our own reflection on the relationship between innovation and tradition or, to use sixteenth-century terms, invention and imitation.

1. MATHEMATICS IN HISTORY AND SIXTEENTH-CENTURY HUMANISM

In the last few years I have been working on the connections between humanist rhetoric, dialectics and the teaching of the liberal arts on the one hand and the developments occurring in sixteenth-century algebra on the other[2].

I have become convinced that many important discoveries of sixteenth-century algebra grew out of other traditions of that era. No matter how translatable these results may be in our mathematics, they remain connected not only to the mathematical tradition but also to other contemporary intellectual traditions and practices. These links are much more than merely stylistic; I am not just suggesting that sixteenth-century mathematics is expressed in a style that is related to sixteenth-century scientific and non-scientific literature. So much is obvious, in any case, and would be admitted even within the field of the history of mathematics. I am suggesting a deeper connection. Mathematicians saw their inventions as contributions to mathematics when they were also consciously transferring to algebra results, meanings and strategies of conceptualization from other disciplines. The perspective of the time was crucial in determining mathematical innovation, i.e. the actual mathematical results, because it promoted explicit "contaminations" with the disciplines of text[3]. These contaminations, typical of the French context, transformed

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algebra after its importation from Italy (and the German countries), so that it became what we call symbolic algebra[4]. Thus, it is this wider contemporary context that is the best depositary of the meaning of those results, as opposed to the mathematical, purely disciplinary tradition alone. Furthermore, there has never been a unitary mathematical tradition, but rather a vast field of previous mathematical activity within which the various schools have carved out their own versions of the past[5].

To look at sixteenth-century algebra in this way necessitates a reflection on the relationship between innovation and tradition or, to use sixteenth-century terms, invention and imitation. For sixteenth-century authors, scientific as well as literary, were often very aware of the need for this kind of reflection.

First, I will discuss how symbolic algebra as we know it grows out of mathematics within humanistic culture, and particularly out the interaction between mathematics and the disciplines of the text. Secondly, I will look more closely at the way in which one sixteenth-century author theorized mathematical creation in "literary" terms, as invention within imitation.

2. THE ALGEBRAIC *ORATIO*

For the humanists, any teaching was intended to prepare the future *orator*. This was the very meaning of humanistic education, according to authorities like Cicero and Quintilian. For those who took Horace as their main authority, the stress was not on the *orator*, but on the *poeta*. So, while *oratio* was the general term indicating structured prose, and often meant "text", the *ars poetica* concerned not just poetry but any kind of oral and written communication[6]: the term applying to both prose and poetry was *opus*. In any event, from a humanistic point of view, any work of science, medicine, law, had to meet certain organizational criteria as a text. While the existence of these common criteria is not unique, the role they had is. It is the case that for humanists what had been a crucial feature of science according to Aristotle, its logic, was entirely embedded in these organizational textual criteria, given the vast program of integration of rhetoric and logic started at the end of the fifteenth century[7].

Another level of research was linguistic. At that time more and more people, including scientists, were learning classical Latin and studying classical texts in order to write in neo-Latin as a second language. Another tendency was "vernacular humanism", encouraging the development of literature, including scientific literature, in the vernacular. Both these lines of research heightened the need of conscious choices in style on the writing of science[8].

Within this general and multifarious late humanistic movement, we shall focus on the French algebraic tradition, i.e. that group of French humanists who imported Italian and German algebra and transformed it into a new humanistic discipline: these are Jacques Peletier du Mans, Pierre de la Ramée, Jean Borrel, Guillaume Gosselin. For this group, the *form* and the *matter* of the algebraic text are identified with one another, insofar as algebra is seen not only as a doctrine to be presented, but also as a formal tool to organize reasoning. Algebra not only *used* rhetoric in order to be constituted into a discipline, but *replaced* rhetoric to shape the new kind of scientific *oratio* or text. This later development would lead us too far, and it is not my intention to develop it here[9]. Rather, I want to look briefly at two of the places in which algebra was connected to texts and to text theories: in the writing of algebraic books and in the theorizing of the evolution of mathematics. The author to be considered will be Jacques Peletier du Mans. He was a well known sixteenth-century scientific writer, particularly in algebra, and also an eloquent author and poet who went so far as committing his theories and practices to a treatise, the *Art poétique*[10].

The two questions I asked earlier take now a more precise form which is the following. In the first part of this paper we shall see how Jacques Peletier du Mans, in his 1554 *L'Algèbre*, develops a new approach to a certain kind of algebraic problem, while insisting that the new solution is a change in the *disposition* of the art, i.e. a change in the rhetorical structure in a sense that we shall see further.

In the second part of this paper, we shall take into consideration Peletier's theory of texts, which is less technical mathematically, but just as technical in poetic theory and dialectics.

3. PELETIER'S ALGEBRA: *INVENTIO*

Peletier published *L'Algèbre* in Lyon in 1554 (Jean de Tournes), fifteen years after the publication of Cardano's *Practica Arithmeticae* (Milano, 1539) and nine years after the publication of the latter's *Ars magna* (Nürnberg, 1545). One might say that Peletier's algebra derives from Cardano and from Stifel, author of *Arithmetica integra*, appeared in Nürnberg already in 1543. For Peletier gives his text the same structure as Stifel's work, although in a much reduced form, while the most important difference is that Peletier's book is entirely devoted to algebra, like Cardano's *Ars Magna*. In fact it is the first printed book on algebra in French and the richest among vernacular books on algebra. Now, we must remember that the selection of topics in any oral or written *oratio* was defined, in dialectics, by the so-called *invention*, whereas *disposition* (or *collocation*) is related to the notion of judgment and order, i.e. the reasoning developed by ordering the "invented" topics. For, all these terms were used in the sixteenth century to indicate specifically the *argumentation* present in the *oratio*: the Orator must build his reasoning starting from the *loci* obtained by the invention, while the construction of the structure of the argument is called disposition. In fact, in the work that we shall examine in the last section, *L'Art poétique*—published a year after *l'Algèbre* and by the same printer—Peletier has different definitions of invention and disposition, more apt to the case of poetry, although poetry is to be taken in its larger sense, as literary work on general topics. "Invention est un dessein provenant de l'imagination de l'entendement, pour parvenir à notre fin. Disposition, est une ordonnance et agencement des choses inventées." (*L'Art poétique*, p. 19)

Going back to *L'Algèbre*, if we describe in some detail the list of topics treated by Peletier, we intend not only to give a sense of the content of Peletier's book, but also to suggest the selection Peletier made among the major algebraic techniques and concepts according to a general plan. In fact, the very specialization of Peletier's book allowed him to emphasize both the techniques, e.g., calculations of monomials, and the concepts, e.g. what he defines as the two main notions of algebra, equations and extractions of roots of binomials, which correspond, for us, to the solutions of first and second degree equations. The title of the section on equations is, significantly, *De l'Equation, partie essentielle de l'Algèbre*. Peletier writes:

> L'équation et l'Extraction de Racines, sont deux parties de l'Algèbre, en lesquelles consiste toute la consommation de l'Art. (...) équation donc, est une égalité de valeur, entre nombres diversement denommés. Comme quand nous disons 1 Ecu valoir 46 Sous. (*L'Algèbre*, p. 22)[11]

But an equation is built on a problem or, in Peletier's terms, on a question, by identifying unknown numbers through known numbers. To put a problem in the form of an equation is a particular technique:

> Premièrement, il s'entend assez, que les nombres exprimés en Questions sont ceux qui nous guident: et par l'aide desquels nous decouvrons les Nombres inconnus. Il faut donc en cette Question proposée, que par le moyen de 46, Nombre exprimé, se trouve celui que je demande. (*ibidem*)[12]

A little further, Peletier gives "*la grande règle générale de l'algèbre*":

> Au lieu du Nombre inconnu que vous cherchez, mettez 1 R : Avec laquelle faites votre discours selon la formalité de la Question proposée: tant qu'ayez trouvé une Equation convenable, et icelle réduite si besoin est. Puis, par le Nombre du signe majeur Cossique, divisez la partie à lui égalée: ou en tirez la Racine telle que montre le Signe. E le Quotient qui proviendra (si La Division suffit) ou la Racine (si l'extraction est nécessaire) sera le Nombre que vous cherchez. (Peletier J. 1554, p. 46)[13]

We have to assign a symbol to the unknown number, then capture the *form* of the question, i.e. to interpret the problem in function of the unknown and its powers, and set the problem as an equation, then modify the latter, for instance by a reduction, in order to obtain a "good" equation. To interpret this passage in our terms[14], we should remember that a "good" equation, according to Peletier, is for instance, in symbolic algebra, of the form $x = c$ for the first degree and $x^2 = bx + c$ for the second, i.e. it has a coefficient for the highest power equal to 1. Therefore, if one starts with an equation of the form $ax = c$, or $ax^2 = bx + c$, the procedure is to divide the second member by a. In the first case we are done, in the second case we obtain the value of x through an algorithm called by Peletier "extraction of the root of the cossic number"[15]. We can start with any sort of question; the art is to find an equation expressing it, and transform it until it becomes a "good" equation, i.e. an equation whose solution formula is known. For instance, we should first operate algebraic sum and possible divisions. Peletier is the first among his contemporaries to have insisted on this part of algebraic theory. The operation by which a problem is given good form and expressed as an equation, from then on was considered typical of algebraic art of thinking.[16]

Peletier defines algebra as dialectics, i.e. as the art of reasoning. This appears clearly in another passage of the same text, where the goal of algebra is to teach one how to think, *à discourir*:

> L'algèbre est un art de parfaitement et précisément nombrer: et de soudre toutes questions Arithmétiques et Géométriques de possible solution par nombres Rationaux et Irrationaux. La grande singularité d'elle, consiste en l'invention de toutes sortes de lignes et superficies, où l'aide des nombres rationaux nous défaut. Elle apprend à discourir, et à chercher tous les points nécessaires pour résoudre une difficulté: et montre qu'il n'est chose tant ardue, à laquelle l'esprit ne puisse atteindre, avisant bien les moyens qui y adressent. (Peletier, J. 1554, p. 1)[17]

By developing algebra according to the model of dialectics, Peletier intends not only to simplify demonstrations, but also to establish an art of thinking—algebra thus becoming this true art of thinking. We should notice also another use of the word *invention*, here in connection with the finding of algebraic solutions with geometrical meaning, use which we tend to ascribe to Viète. But, more generally, we see here that Peletier's explicit goal is to represent algebra as the art of finding a presentation of problems that can make it possible to solve them all. Peletier is not the first to use the slogan, again usually attributed to Viète, that algebra teaches us how to solve all problems[18].

4. PROBLEMS IN SEVERAL UNKNOWNS. *DISPOSITIO*

If putting a problem into the form of an equation is considered by Peletier to be an operation in itself, and also the strong point of his method, then solving a problem in several unknowns should take his method to task. Peletier innovates with respect to all his predecessors precisely on this point. In fact, he innovates with respect to Stifel, and does so by making use of Cardano's *Ars Magna*, the most recent major work on algebra. Peletier gives the following definition of second unknowns, which he calls, following Stifel, *secondes racines*:

> Les Racines Secondes viennent en usage quand deux nombres ou plusieurs se proposent, entre lesquels ne se fait aucune comparaison expresse par addition, multiplication, division ou proportion, par différence, ni par Racine: qui sont les cinq manières de comparer les nombres ensembles. Desquelles la proportion est la principale, car les autres seules bien souvent n'excusent pas l'usage des Secondes Racines. (*L'Algèbre*, p. 96)[19]

Now, this passage clearly retraces the lines of a similar passage in chapter XI of the *Ars magna*, and shows us that in fact Peletier also adopted Cardano's theoretical point of view with respect to the second unknown:

> We have been using [two] unknowns, and no relation has been assumed between the two numbers at the beginning, either by way of addition or subtraction, multiplication, division, ratio, or root—for numbers may be related in these five ways. But if one [of these relations] exists, there is no necessity for [using] a second unknown, for the problem can be solved by one unknown. (*Ars Magna*[20] ch. X, p. 79)

We may note in passing that we have here a sixteenth-century version of a definition of functional relation, called *comparatio* in the original, insofar as Cardano mentions all determinate operations in a general way. Of course, the use of the *secundae quantitates incognitae* allows one to perform, through the solution of the system, just those operations that would be indefinite.

Now let us look at Peletier's version of the problem that opens the chapter entitled "*De secunda quantitate incognita*" of Cardano's *Ars magna*.

Cardano had written:

> Up to this point we have been treating of new discoveries quite generally. Now something must be said about certain individual types. It frequently happens that we must solve a given problem by using two unknown quantities. There follows an example of this which we could otherwise explain only with difficulty. Three men have some money. The first man with half the others' would have had 32 *aurei*; the second with one-third the others', 28 *aurei*; and the third with one-fourth the others', 31 *aurei*. How much had each? (*Ars Magna* ch. IX[21])

Cardano is not ready to give a general rule. The next sentence gives the procedure of assignment of the unknowns:[22]

> We let the first unknown thing be the first man's share, the second unknown thing the second man's share; thus for the third man there will be left 31 *aurei* minus one fourth of the thing and one fourth of the quantity. (*ibidem*)

Cardano pursues his calculations on this basis for a couple of pages. As we can see the third unknown is defined in terms of the other two. Peletier repeats Cardano's statement and gives a first version of the solution following Cardano's reasoning, adding only a few explanatory comments. At the end he adds some very inspiring remarks:

> En cet Exemple, j'ai suivi de point en point la proposition et la disposition de Cardan. En quoi j'ai été aussi long comme lui, et un peu plus clair. Et n'eut été pour montrer la singularité de l'Algèbre, et comme elle gît en discours, et comme elle exerce les esprits: j'eusse laissé cette explication sienne, laquelle il appelle facile, pour en mettre une autre qui s'ensuit, de notre dessein. (*L'Algèbre*, p. 110)[23]

Cardano's solution gives Peletier the opportunity to state that algebra is explicit reasoning. For our purpose it is particularly important to notice that Peletier uses the word *disposition* for Cardano's reasoning.

In the next sentence Peletier starts his own solution procedure: "Le premier a 1R. The second, 1A. Le tiers, 1B." According to the first hypothesis[24], $1R + 1/2\,(1A + 1B) = 32$. By reduction and "transposition",

I: $2R + 1A + 1B = 64$. Peletier calls this the *first equation*. According to the second hypothesis, $1A + 1/3\,(1R + 1B) = 28$. Thus,

II: $1R + 1B + 3A = 84$, *second equation*. According to the third hypothesis, $1B + 1/4\,(1R + 1A) = 31$, hence

III: $1R + 1A + 4B = 124$, *third equation*. We now **add the third to the second equation**, and we get the *fourth equation*. i.e.

IV: $2R + 4A + 5B = 208$. We now can **subtract the first equation from the fourth**, and we get the *fifth equation*

V: $3A + 4B = 144$.

We can now take another direction, and **add the first and the second equation**, we get

VI: $3R + 4A + 2B = 148$, which is the *sixth equation*. By **adding the first and the third equation** we get the *seventh equation*

VII: 3R + 2A + 5B = 188, whereas **by adding the sixth and the seventh equation** we get 6R + 6A + 7B = 336, which is the

VIII: *eighth equation*. Let us now multiply the third equation by 6, getting IX: 6R + 6A + 24B = 744, i.e. the *ninth equation*. Given that the first two terms of equation eight and nine are equal, we can write 17B = 408, and we obtain the third number, B = 24.

Now, let us go back to the fifth equation[25], 3A + 4B = 144: by subtraction we get 3A = 144 − 96, we obtain 3A = 48, i.e. A = 16, the second number.

Finally, because of the first equation, 1R + 1/2 (16 + 24) = 32, hence 1R = 32 − 20 = 12, the first number.

Peletier concludes: "*Ce discours est trop plus facile que l'autre. Mais il fait bon voir deux inventions en même intention*." I.e., " This talk is too much easier than the other one. But it is good to see two inventions with the same intention."

This procedure might not look easy or brief to us, but it was remarkably short if compared to Cardano's procedure. What did change? The main innovation is in the introduction of as many symbols as there are unknowns in the problem: here the unknowns in the problem coincide with the unknowns of the equations. Furthermore, Peletier is very systematic in structuring the solution through the various transformations of the first equations, those which, for us, belong to a "system." Then he uses, as Cardano does, the method of addition and subtraction of equations. However, he does this in a different way than Cardano because he does not introduce the arbitrary coefficients which we perceive as artificial and apparently were considered somehow *ad hoc* also at Peletier's time. Peletier's innovations make the solution simpler and shorter or, according to Peletier's stylistic values, "short and clear" and the notation points to the path later taken by Jean Borrel[26], Guillaume Gosselin[27] and then François Viète: the use of a sequence of letters of the alphabet.

What Peletier has done is change a procedure. But he explicitly describes this change as a change in the text. By establishing a specific order for the procedure (immediately represented by ordinal numbers) as well as a general method (attributing different symbols to all the unknowns of the problem) and by transforming the coefficients according to the order established, Peletier intervenes in the typical structuring of algebraic reasoning. He looks at the problem not only as a particular problem but as a pattern, as *suggested by its form*. As it is discussed elsewhere in this volume, this is not specific to sixteenth-century symbolic algebra, but also applies to Mesopotamian and Chinese algebra. What is characteristic here is the form of the solution: not only is the solution procedure given in steps, thereby offering a pattern for the solution of other cases, but also the steps depend on the use of symbols, and are presented in an explicit order denoted by roman numerals. We are therefore in the presence of a text which has been *constructed* as a "second level text," in the same sense as we speak of equations in their general form. Jacob Klein, writing in the first half of last century, has formulated the conjecture that this character of second level text was embedded in the very notion of second order numbers. Klein's point, as I understand it, is that the passage from the "monads" of the Greeks to the more general numbers developed in early modern arithmetic (thanks to the Arabs and to Diophantus) is not a linear development, but a change in perspective. Given that the Renaissance world rests upon the ancient one, modern numbers would be first of all numbers of numbers, i.e. not numbers about the external world but about human tradition of numbers. But I would argue that general numbers are just the counterpart of (in other words, come with) equations and problems in their general form. In fact, maybe the change in problems is more fundamental that the change in numbers. What we have seen is a way in which problems in their general form come from a theoretical concern about the order and the role of the scientific text.

We shall come back to this peculiarity of algebra in the next section. Here, it is important to notice that Peletier considers his innovation in the treatment of problems in several unknowns as a contribution to the *disposition* of algebra:

> Si la Disposition est celle qui donne dignité aux choses, et si la forme est celle qui fait être une chose celle qu'elle est, je me promets de m'être ici tellement acquitté. (*L'Algèbre*, proème, f. 9)[28]

Peletier does not give us here an explanation, but an example of his notion of disposition, and the example is his entire work, *L'Algèbre*.

The necessity to make explicit the grammar (at least the orthography) and the rhetoric implied in his enterprise shows that to write an algebraic text in French and in accordance with the values of contemporary humanistic culture was, in Peletier's own eyes, an entirely new project. He was in fact one of the main promoters of new scientific writing, among authors who were particularly aware of the requirements and potentialities of printing.

Like the ancient *Orator*, the sixteenth-century writer should not miss out on the opportunities offered by his *oratio*[29]. The first function of the rhetorical art in algebra was, according to Peletier, to provide the tools for rendering a text efficient, and the standards by which to judge it. Thus he used rhetorical technical terms to articulate the organizational criteria for texts that did not belong to the classical mathematical tradition.

5. THE TRUE SENSE OF TEXT AND THE CASE OF ALGEBRA. *IMITATIO*

These examples give us an idea of what the practice of *invention* and *disposition* in algebra meant for Peletier: they allowed him as an algebraic author to write his text, but also were part of the art of reasoning that algebra was supposed to supply.

We can now see more closely Peletier's wider theory of texts. Here appears another quite original contribution of Peletier's thought, which at the same time shows that his humanistic culture gave him an excellent standpoint for a sophisticated view of the matter. Peletier's work *L'Art poétique*, published in Lyon in 1555, a year after *L'Algèbre*, is entirely centered on the notion of *imitatio*. In clarifying his outlook about the imitation of the classics, Peletier makes explicit a theory of the development of scientific disciplines. Already in *L'Algèbre* Peletier gave an account of the history of algebra in terms of a cumulative process, where invention does not start with a person, but with a people. According to his theory, each language is the depositary of the creativity of a people, and is apt to develop the notions which have been the most familiar to that particular people, such as the juridical terms for Latin. In this way, a people is disposed to develop an art further. Authors intervene at this level, and they can be considered inventors because they play the crucial role of transmitting knowledge. What an author does, says Peletier, is to give newness to old things and authority to new things. As Peletier writes in a long section on imitation,

> L'office d'un Poète est de donner nouveauté aux choses vieilles, autorité aux nouvelles, beauté aux rudes, lumière aux obscures, foi aux douteuses. (*L'Art Poétique*, p. 24)[30]

Peletier is elaborating on Horace's theory of imitation: each author extracts a matter (*materia*: a topic, an idea) from the common knowledge and gives it an original status[31]. For Peletier, this practice of transmission is explicitly connected to the invention of new words[32]. First of all, the stake is the creation of a scientific French. But, more generally, in order to fulfill his task the poet must appropriate "old things" to his language, i.e. to translate them to the new language and to the new time. A people is the creator of an art, and its language its depository, but the plasticity of language makes it possible for a people to introduce new words first to translate from another language then, with luck, practice and new ideas, to create a new art.

This view establishes a chain of authors who are readers of previous authors, where not only imitation is always creation, but any creation is also a translation and an imitation. In our words, a text is a function of the reader, and if the reader is in turn an author, the text will be transformed into another text. There is a use of the text, and this is particularly important not only because it illuminates what scientific creation and invention meant for Peletier, but also because, in the case of algebra, the reader is supposed to be active and creative, is supposed to *use* a procedure applying it to different cases, in fact is supposed to transform a problem into an equation which will yield the solution.

This doctrine by Peletier is meaningful for us in two ways: firstly, it establishes Peletier at the origin of the French algebraic tradition, for after Peletier, Guillaume Gosselin in 1577 and 1583, then Viète in 1591 and Descartes, to mention only the most significant, took the same direction as Peletier, writing texts that are highly sophisticated in terms of language, notation and organization of the material. Algebra became with them an important Latin discipline, whereas, according to them, it had been nothing but an obscure technical calculation in Arabic and vernacular languages. Viète and Descartes neglected to underline their dependence even on their most recent sources.

Secondly, Peletier's lead was followed by later authors insofar as they adopted a similar invention and disposition in their books of algebra, but also stressed the role of algebra as a method, as an art of discovery or of invention, and finally, in Descartes, as the basis for *mathesis universalis*. Algebra as a second level text was born in these writings.

What Peletier did not manage to transmit to his successors, was his concept of transmission, his sense of text. He saw texts as belonging to a chain of texts. For history can be constructed and reconstructed, and this may lead to skepticism or relativism. But the texts exist. Insofar as they are well-built they are a perennial challenge to imitation, the only bridge, though unstable, to the understanding of the past.

I would like to conclude with a passage from Peletier's Latin translation of his *L'Algèbre*[33]:

> Those who have thought a bit more deeply about the beginning of things do not ascribe the invention of disciplines to a single person, but acknowledge that it is for the germs of disciplines as for the virtues located in souls and for the sparks that the eternal spirit

> lights in us, that is to say, that they emerge from a common genius. The destiny of things through time is controversial: destiny, through its vicissitudes, offers such fecundity of things that those who take advantage of present beauty without maintaining any deeper memory not only refer everything they receive to their century but also can not conceive of any transformation in the future. This calamity occurs in an era that we know all too well: in people's minds there is no expectation of betterment. And there where once the arts flourished, people imagine that they are not being reborn but starting from scratch. Because of the perpetual vicissitudes of things, those who do not have a broad outlook consider as unique to the present anything that they did not receive directly from the hands of the ancestors. (Peletier J., 1560, preface).

With this passage, Jacques Peletier stated some of the questions of philosophy of history and history of science that we are now trying to face, such as a philosophy of history which can include a notion of the past and of the development of knowledge. Other historians of the Renaissance have stressed the fact that sixteenth-century people outlined problems of language, interpretation and history that we are still elaborating on: I am thinking of works as different as Donald Kelley's, Richard Waswo's and Ian Maclean's[34]. It has become acceptable, in intellectual history, to state these similarities as an attempt to make explicit the intellectual expectations inspiring our interpretation of ancient texts. However, in the history of science, we are accustomed to oppose innovation and tradition. With Peletier we can see that the invention of a work of art or science does not occur in opposition to the past, but grows out of it, through imitation. His theories make us aware of the necessity to refine our knowledge of the techniques of writing and imitating used in a more or less conscious way by mathematicians of the past.

NOTES

*1 I wish to thank Hannah Davis Taïeb for her precious help in the process of transforming my original paper into an English text.

2 For a more in-depth discussion of these themes, see (Cifoletti G.C. 1992) forthcoming as a book. A global presentation of the French algebraic tradition in the sixteenth-century appeared in (Cifoletti G.C. 1995).

3 By this name I refer to the *artes* known in the Middle Ages as *sermocinales*, the arts of discourse, that is the set of the three arts of the trivium, grammar, rhetoric and dialectic or logic, once they got deeply modified by the humanistic reform. In fact, Lorenzo Valla and other humanists could have collected them under a similar heading for two main reasons: because of the addition of philology and historical studies and because the role of the *orator* had as much to do with writing as with speaking.

4 I call symbolic algebra: 1) a symbolism that allows the treatment of general equations, which is to say different letters for the unknowns on the one hand and the coefficients (or known terms) on the other; 2) the operations with monomials; 3) the determination of solution formulas for equations of the third and fourth degree. And then further, with respect to the theory of equations: 4) the elaboration of techniques for the reduction of equations to some standard cases; 5) the determination of relations between coefficients and roots; 6) the determination of number of roots. All these aspects were not treated in an exhaustive way, but at least partially studied much before the sixteenth-century, in the Arabic texts, and even Descartes and Fermat did not reach exhaustiveness on all the points.

5 This creation of genealogy is crucial to the self-definition of each school with respect to the others. For the foundation of a European tradition of mathematics, see Goldstein, J. Gray, J. Ritter (eds.) 1996.

6 From the rich literature on this topic, I will cite only Gordon, A.L., 1970.

7 Again the choice of works on this topic is great, among them: Jardine, L. 1983, as well as K. Meerhoff, K. 1986. My argument here is somewhat similar to what Lisa Jardine developed with respect to Francis Bacon in Jardine, L. 1974.

[8] For the transformation of scientific writing in the Renaissance, in connection with the development of Neo-Latin, see for instance the essays by A. Blair, I. Pantin and J. Peiffer in: Chartier, R., Corsi, P. (eds.) 1996: 21–42, 43–58 and 79–93. On the question of language in France, see Castor G. and Cave T. 1984.

[9] I have developed this point in (Cifoletti, G.C. 1995), and, in connection with Descartes, in chapter 7 of (Cifoletti G. C. 1992). For, Descartes' *Regulae* are a very accomplished version of the sixteenth-century dream of proposing algebra as the new rhetoric. Algebra, here, is at the same time the method of solution of equations and a form of symbolic calculation concerning equalities. On Descartes' dependence on the French algebraic tradition, see also Cifoletti G.C. 1998.

[10] *L'Art Poétique*, Lyon, Jean de Tournes, 1555.

[11] Peletier was an innovator and a theoretician of texts at yet another level: orthography. I have decided to keep it as much as possible in this first quotation, so that the reader will notice some of the peculiarities of Peletier's orthographic rules. My translation is: "Equation and root extraction are two parts of algebra, in which consists all the refinement of the art. (. . .) Therefore, the equation is an equality of values between two numbers denominated differently. As when we say 1 pound is worth 46 pence."

[12] "First, it is clear that numbers expressed in questions are what guides us, by the help of which we discover unknown numbers. In the question asked here, we must find the number asked by means of 46, number expressed."

[13] My translation is: "Instead of the unknown number you are looking for, put 1 R: by means of this develop your reasoning according to the formality of the question asked, until you have gotten an appropriate equation, reduced if necessary. Then, divide the part put equal to it by the number of the highest cossic sign [a cossic sign being the unknown and its powers, cossic algebra being the four operations and extractions of roots applied to cossic numbers or monomials, my remark], or extract the root according to the sign. And the quotient resulting from this (if division is enough) or its root (if the extraction is necessary) will be the number you are looking for."

[14] Peletier does not use letters for the coefficients, which are taken as positive. Furthermore, he deals almost exclusively with first and second degree equations and positive roots.

[15] In this case, we get the whole second member (a binomial) as a power, of which we extract the square root. This algorithm gives rise to the solution formulas for second degree equations, very similar to the procedures present in the Mesopotamian tradition.

[16] Descartes' *Regulae* is a major later example.

[17] "Algebra is an art of numbering in a perfect and precise way, and of solving all the algebraic and geometrical questions which have a possible solution by rational or irrational numbers. Its great singularity is in the invention [finding] of all sort of lines and surfaces, where the help of rational numbers is missing. It teaches how to reason, and to look for all the points necessary to solve a difficulty: it shows that there is nothing so arduous that the mind cannot reach it, if we decide properly the means useful for it." (My translation.)

[18] It was in fact a *topos* of algebra. Aside from its presence in the Arabic tradition, one should notice that more recently Cardano had made use of it in his *Practica Arithmeticae*. It is therefore particularly interesting to see the same *topos* transformed in the new context.

[19] "Second roots are used when two numbers—or more—are proposed, between which there is no explicit comparison by addition, multiplication, division or proportion, nor by difference or root, which are the five ways of comparing numbers among themselves. The proportion is the most important of these, because the others very often do not justify the use of second roots." (My translation.)

[20] We use here the English translation (Cardano, G. [1545] 1968: 71).

[21] See also Girolamo Cardano [1663] 1966, IV:241.

[22] Here I translate from Latin, because Witmer adopts his own notation directly at this point, and this would obscure matters in our context.

[23] "In this example, I followed point by point Cardano's proposition and disposition. By doing this, I have been as long as him, and a bit clearer. If it had not been for showing the singularity of algebra, and how it consists of reasoning and how it exercises the mind, I would not have mentioned this explication of his, which he calls easy, to replace it by another, the following, which is mine." (My translation).

[24] Here I shall adopt, unlike Peletier, our signs for $+$, $-$ and $=$, where he has p., m. and *égal*. I also introduce the brackets, for typographical convenience.

[25] Peletier writes "3A p 3B étaient égales à 144": the second term is a misprint and should be 4B; the next passage does not carry over the mistake.

[26] Jean Borrel, also known as Buteo, was another Renaissance mathematician devoted to the revival of the classics. He was the author of another algebraic text, the *Logistica* (Buteo 1559), and had a theoretical dispute with Jacques Peletier on the latter's translation of Euclid.

[27] Gosselin published three mathematical works, all related to algebra, the most important of which was his *De Arte magna* (Gosselin 1577). His project, only partially realized, was to recover Diophantus' *Arithmetic* in the terms of the most recent developments in arithmetic, i.e. algebra. Gosselin translates Tartaglia but does not mention Bombelli.

[28] "If Disposition is what gives dignity to things, and if the form is what makes a thing what it is, I consider I have done my duty." (My translation.).

[29] For the discussion of the possibilities of printing by Erasmus, see J. Chomarat 1981, 387–393.

[30] "The duty of a poet is to give novelty to old things, authority to new ones, beauty to the rough, light to the obscure, trust to the uncertain." (My translation).

[31] See Jacques Peletier du Mans 1545, f. 11v.

[32] "Quant à l'innovation des mots, faudra aviser si notre Langue en aura faute: et en tel cas, ne se faut feindre d'en former de nouveaux. Un mot bien déduit du latin aura bonne grâce, en lui donnant la teinture françoise." (*ibidem*, p. 37) i.e., in my translation: "As to innovation in words, it will be necessary to see whether our language will lack of them: in that case, it is not necessary to form new ones. A word well derived from Latin will be fine, if one gives it the French color".

[33] Peletier's Latin version has a few variants, including the entire preface. This is my translation.

[34] See these works also as an illustration of contemporary understandings of sixteenth-century's "disciplines of the text": Donald R. Kelley 1970; Richard Waswo 1987; Ian Maclean 1992. Waswo's can be considered the most radical understanding: one of his "principal theses" is that some very serious doubts about the dualistic model (correspondence between discrete signs and discrete ideas), along with alternatives to it, became articulate and well diffused in the culture of fifteenth and sixteenth centuries.

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