Chapter 8
Mathematical Progress or Mathematical Teaching? Bilingualism and Printing in European Renaissance Mathematics

Giovanna C. Cifoletti

In mathematical books of sixteenth-century Europe the question of teaching is present throughout. The very need for mathematical books is presented as the need of explaining mathematical contents effectively and of training the readers in mathematical techniques. The book should make understanding and learning mathematics more accessible: while for most books their printing was justified by being spiritually and morally edifying, for mathematical books the explicit purpose was to make mathematics clearer to follow and easier to absorb by a larger group of people. Mathematical books were intended to make the task of teaching lighter and even superfluous, providing the basis for self-teaching. These are the statements outlining the explicit principles of the actors. These statements open some questions as to the actual use of these books. The more basic question is which kind of teaching, preceptorial, private or public, was available at that time for these subjects, the following is in what teaching situation the books were present and used: here I shall work on the interplay between bilingualism and printing.

The examples will be some arithmetical and algebraic books which seem to give more direct information concerning the teaching situations. In particular I shall look at Gemma Frisius' *Arithmeticae practicae methodus facilis* (1540)[1] Oronce Finé's *La Théorique des Ciels, mouvements, et termes practiques des septs planètes* (1528), *Les Canons et documents très amples, touchants l'usage et practique des communs Almanachz que l'on nomme ephemerides. Briefve et isagogique introduction sur la judiciaire astrologie* (1543), and the posthumous *La composition et usage du*

[1] Frisius (1540).

G.C. Cifoletti (✉)
Directrice d'études à l'Ecole des Hautes Etudes en Sciences Sociales,
Centre Alexandre Koyré, 27, rue Damesme 75013, Paris
e-mail: cifolet@gmail.com

A. Bernard and C. Proust (eds.), *Scientific Sources and Teaching Contexts Throughout History: Problems and Perspectives*, Boston Studies in the Philosophy and History of Science 301, DOI 10.1007/978-94-007-5122-4_8,

quarré géométrique, par lequel on peut mesurer fidèlement toutes les longueurs, hauteurs et profunditez (1556). By Jacques Peletier du Mans we shall examine his *Aritmetique* (1549),[2] *L'Algebre* (1554)[3] and *De Occulta parte numerorum quae algebra et almucabala vulgo dicitur* (1560),[4] *De Usu geometriae* (1572) and *De l'usage de la Géométrie* (1573), finally Pedro Nunes' *Libro de Algebra* (1567).[5] Notice, however, that these books will be taken into consideration here in different ways: as examples of textbooks in public or in private teaching, as sources for facts concerning teaching and learning, as reading of sixteenth-century theories about mathematical teaching and learning. I hope this could be taken as a contribution to refine our future questions about the connection between scientific books and scientific teaching in early modern Europe.

8.1 Gemma Frisius' *Arithmeticae Practicae Methodus Facilis*. The Integration of Abacus Mathematics with *Algorismus* at the Colleges

Gemma Frisius (1508–1555) had an impact on the history of arithmetic and algebra because of his work on arithmetic: *Arithmeticae practicae methodus facilis*.[6] This book is in Latin and is explicitly addressed to college teaching. Textbooks for college teaching were traditionally published in Latin, so that, in general, the translation into Latin indicates, in France, the passage of a book to college teaching.

It has been the most popular Latin arithmetic of the time, with about 74 editions at the end of the sixteenth-century. At first, its format made it belong to the genre of the *algorismus,* though adapted to the taste of the day. *Algorismus,* or university practical arithmetic, was the medieval genre explaining the use of indo-Arabic numerals, mostly meant to teach elementary arithmetic (with indo-arabic numerals) as a basis for astronomy and cosmography. It was in Latin, the humanistic neo-Latin, namely a style that revived the models of classical Latin and addressed to college students as a prerequisite to university's teaching. It contained traditionally a definition of numbers and digits, the four operations with integers, the four operations with decimal fractions, the four operations with sexagesimal fractions. The last section was meant to instruct the students to use astronomical tables. This was crucial because future physicians could then determine horoscopes, whereas others could know simply measure time or the longitude. But Gemma Frisius casts the topic of arithmetic and its four operations in an entirely different framework.

[2] Peletier (1549).

[3] Peletier (1554).

[4] Peletier (1560).

[5] Nunez (1567).

[6] Frisius (1540) in this connection, see (Van Ortroy 1920; Echalier Serra 2011). On Gemma Frisius and his bibliography, see (Hallyn 2008).

At folio 5 we read:

> There are four sorts of arithmetic and by means of them all the rules and almost all questions are solved. We call species certain forms of operation by means of numbers: as in Dialectic the forms of the arguments are included in four species, i.e. syllogism, induction, enthymeme and example. The first of these is addition.[7]

We recognize here the reference to the teaching situation of the colleges, and even not to any kind of college, but to humanistic colleges[8]: one of the main features of humanistic education was the rhetorical reform of logic[9] which, precisely, stressed the role of *example* and *enthymeme*, which are the rhetorical versions of the logical notions induction and syllogism: the first indicates the instance, the case, whereas the latter indicates probable syllogisms. The book will be about the four operations, in analogy with the four sorts of reasoning, applied to different sorts of numbers. This presentation will be crowned by the rules based on the rule of three, presented as the theory of proportions applied to commerce.

This short book (69 folios in 8°) accomplished in a concise way what had been Pacioli's program: integration of abacus schools' arithmetic with university's *algorismus.*[10] Its language and structure makes it fit into the genre of *algorismi*, but it goes farther than contemporary books of the genre in the treatment of the rules of commerce and in the implicit use of algebra. In this sense it is a proper introduction of the mathematics of business in the context of university arithmetic.[11]

After the four operations on integers and geometric progressions, Frisius introduces "the rule of proportions, or of the three numbers". This is a very interesting mixture of abacus schools' mathematics of commercial rules with the theory of proportions. The third part, according to the genre of the *algorismi*, would have normally been about astronomical or sexagesimal fractions, the two adjectives being used as synonymous. Instead, it is about the rules of commerce: "De Regulis vulgaribus". Gemma Frisius deals with some variants of the rule of company, of the rule of alloys and then at length with the rule of false position. There are three sections corresponding to the three rules of the Thing or of algebra, which give

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[7]Quatuor omnino sunt Arithmetices species, per quas omnes regulae quaestionesque omnes fere perficiuntur: vocamus autem species certas operandi per numeros formas, quemadmodum in dialectice argumentorum formae, quatuor comprehenduntur speciebus, syllogismo scilicet, inductione, enthimemate et exemplo. Prima harum est Additio. (Frisius 1540, f. 3v). All translations are mine.

[8]By this I mean the colleges developed following the pedagogical prescriptions present in new schools of late fifteenth and early sixteenth centuries Italy and Northern Europe. See (Grafton and Jardine 1987).

[9]I mean the transformation of logic which culminated with Petrus Ramus's *Dialectique.* See (Cifoletti 2004, 2006c).

[10]For these schools and for *algorismi* and *abacus* traditions, see the Chaps. 6 and 9 by S. Lamassé et E. Caianiello.

[11]The importance of this innovative feature is stressed in (Zemon Davis 1960). As to the difference between "abacus" and "abbacus", I take it as a variant without meaning, given the instability of Italian spelling: I therefore follow Caianiello not accepting Van Egmond convention.

three cases which in our terminology would be ascribed to equations up to the third degree, followed by the extraction of square and cubic roots.

Notice however that Gemma Frisius' book does not contain any algebraic notation or symbol for the unknown quantity, therefore we cannot talk about equations. Gemma Frisius explicitly says that it would be much easier to deal with these questions by algebra, but given that this is a whole topic of its own and that he hopes to write a treatise about it, he omits algebra for now. Yet, he calls 'rules of algebra' some applications of the rule of false. The rule of false was a procedure at least as old as Babylonian mathematics. The rule says: in a problem expressible in terms of one thing, let us insert an arbitrary value (the false position). Then the expression of the problem in that value will be false, but proportional to the expression of the problem in the value sought, so that we can apply the rule of three again.

The example given by Frisius here is the classical problem of three men, which in our notation would be a system of three equations, in which we use the method of substitution.

Having the algebraic technique in mind allows Gemma Frisius to introduce some algebraic arguments without an algebraic notation, that is, equations with unknowns. In fact, he worked on examples in which the application of the rule amounts to the extraction of the square root. This simple technique could make the readers of Frisius' arithmetic open to the use of solution formulas for algebraic equations. In fact, Michael Stifel in 1544 will take the same move. In folio 31 Frisius explains why he prefers to arrange all the matter by focusing on the *regula falsi*, without dealing with algebra proper. He declares that he is aware of the fact that algebra would provide a more certain and by far easier procedure, because nobody has seen a mathematical art more excellent and elegant. However, many have written about it[12] and he also plans to write an algebraic treatise himself.

Gemma Frisius' book continues with the section on the rule of algebra in fractions, which is relatively unusual. The fourth part is again on proportions, but here Frisius deals with mean proportionals. Gemma Frisius proposes a constant translation between the theory of proportions and commercial rules. What is most striking is that in the 1547 edition Frisius adds an absolute novelty: at this point he introduces a section on usury. It is noteworthy that this section was added by Gemma only in the 1547 Antwerp edition,[13] which was the fourth edition: this was after the publication of Peletier's Parisian edition. Frisius develops both simple and compound interest.

With respect to his notable omission, astronomical fractions, Frisius explicits his view point in the second edition, where he reintroduces astronomical fractions in an appendix. He argues that it is not difficult to deal with them and relegates the topic to an appendix: a significant change in the structure of the genre, but also in the conception of the book. Furthermore, it is not what one would expect now from an

[12]He only mentions Christoff Rudolff's *Die Coss,* published in Strasbourg in 1525.

[13]Frisius (1547).

astronomer of renown. Yet, the book was hugely successful, so apparently it did not miss its target. We have perhaps an indication of a reason for this in the words by which Frisius introduces his appendix:

> I don't see any important difficulty in physical or astronomical fractions, so that in order to make easier to young people the road to excellent disciplines, for which especially we meant to help the reader, I shall note some points which can appear difficult.[14]

Frisius suggests that of course astronomy and physics are the interesting applications of arithmetic, but once practical arithmetic in his new sense is absorbed, together with the rule of the Thing, sexagesimal fractions will be easily manageable.

This book is important in the history of mathematics not only because of its content, but also because it is an extraordinary bestselling textbook. Furthermore, the various editions included many corrections and additions. Special attention should be given, first of all, to the additions during Frisius life. So, if we take into consideration the editions until 1555, additions consist of references or of entire sections concerning extra topics. The references are to classical sources, such as Euclid's *Elements* and Ptolemy's *Almagest*. The main topic added is usury, included by Frisius himself in 1547. As to the later authors, the more substantial additions have been introduced by Peletier. Jacques Peletier du Mans (1517–1582) published an annotated edition of Frisius's text already in 1545.[15] His additions are *Compendium de fractionibus astronomicis*, devoted to sexagesimal fractions, traditionally included in the *algorismi*, and *De Cognoscendis per memoriam calendis*.

This change introduced by Peletier indicates that the book was used in the universities, where the new topics introduced by Frisius, as well as his style of presentation, were appreciated, but also where astronomical fractions were taught, so that a written presentation was needed. The other additions seem to be of interest for a larger audience, dealing with the basics for calendars and almanacs: *De cognoscendis per memoriam Calendis, Idibus, Nonis, Aureo numero, et loco Solis et Lunae in Zodiaco*. It contains the basic tools for Latin calendars and horoscopes. Peletier added also a short section on algebra: in fact he gives the proofs for the extraction of square and cubic roots, or rather transformed into demonstrations the intuitive arguments Gemma Frisius had provided for the algebra section, but again without an algebraic notation. This edition of Frisius' book was subsequently reprinted several times, and was in fact one of the great publishing successes of its time. The editor and translator of the Italian edition, the professor of rhetoric Orazio Toscanella, was particularly interested in reproducing Peletier's contribution on astronomical fractions, which he praised profusely in his introduction.[16] Pierre Forcadel, who became *lecteur royal ès mathématiques* in Paris, provided a French

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[14]Frisius (1547, fol. 64r).

[15]Frisius (1545). This first edition published by Richard was followed by a 1563 edition published by Cavellat and a 1572 edition published by Marnef.

[16]See (Frisius 1567).

translation of Frisius's arithmetic: *L'Arithmétique de Gemma Phrison*.[17] It is important to notice that both French editors were former students of a *lecteur royal ès mathématiques*, Oronce Finé.[18]

In conclusion, Gemma Frisius managed to produce a compact textbook for colleges of Northern Europe. In terms of the mathematical content, it was sensitive to the new humanistic pedagogical reform, combining abacus mathematics with the arithmetic of *algorismi*, and doing this in a "short and clear" way, according to the rhetorical reform. Also, it was in Latin because it was addressed to colleges, but his language followed the rules for humanistic neo-Latin. His initial choice of inclusion of commercial rules was successful, and was soon confirmed by the inclusion of sensitive topics such as usury. However, Frisius' decision to understate the role of astronomical calculations was soon corrected: in following editions and Peletier's addition in this matter was highly appreciated.

8.2 Bilingualism and the French Vernacular Scientific Book Project: The Pedagogical Programs of Finé and Peletier

8.2.1 Finé

We have seen, so far, an example of innovation in genre coming in Latin scientific literature. In fact, two new genre of text appeared in this period, which can be seen as intermediate with respect to previous genres: some, like Frisius's *Arithmetica*, included "vernacular" material in a university textbook in Latin, others, like some new vernacular texts, gave a classical, literate shape to vernacular material. The latter genre took explicitly some distance from university teaching while shaping these books. However, if the book got some success, it was not unusual that the author himself translated these books into Latin in order to introduce the scientific content, but also the new presentation, in university teaching.

But things could also go the other way, so that university professor felt committed to publish in the vernacular, so again the same author published his book first in Latin and then in the vernacular. In fact many university professors were strongly engaged, at the time, in the promotion of scientific literature in the vernacular. First, this was connected to the general tendency in humanistic culture, the so called "*umanesimo volgare*". The larger program was to create a new encyclopedia in the vernacular. For mathematical sciences, this was also connected to the non-academic origin of the disciplines, often transmitted from Arabic through abacus schools.

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[17]Published in Anvers by Jean Withage, in 1584.

[18]Oronce Finé (1494–1555) was a contemporary and fellow cosmographer of Gemma Frisius, teaching mathematics at the Collège Royal from 1530 to 1555.

There is also a more general aspect which should be taken into account: in fact, what we see is that both the authors and the readership of sixteenth-century books are mostly bilingual and often could have access to both versions of the books. The choice was mostly connected to a different *reading context*, so that paradoxically different readerships could consist partially of the same people. With this in mind, looking at the choice of language is still a strong indication of the teaching context.

Oronce Finé was the first in France to launch the program completed by the next generation of mathematicians, namely to publish in French a new genre of mathematical book, in a new style, the French vernacular scientific book.

This new genre took the form of craftman's manuals rather than that of textbooks connected to teaching practice, and was aligned with the tradition of arts and crafts in the vernacular. Manuals could coincide with textbooks, but were also conceived of as a distinct genre: by and large, manuals are connected to abacus schools and *Rechenmeister*'s shops, whereas textbooks are related to colleges. The French vernacular scientific book project is of interest to us precisely because the authors started publishing these books in the vernacular, as a record of the teaching which was going on in the vernacular teaching situations, whereas textbooks took as models the Latin medieval texts. Oronce Finé published three works in French. He picked his topics both from the Latin tradition among university disciplines and from the vernacular tradition outside the university. The purpose was for him, as for many later sixteenth-century French authors, to launch a new literature which followed new criteria, according to the taste of the day.

To the first category belong two books, published in 1528, was *La Théorique des Ciels, mouvements, et termes practiques des septs planètes*. It was a version of Peurbach's *Theorica Planetarum*, who had innovated in the genre[19] inaugurated by the medieval text attributed to Gerardus of Cremona, the Latin *Theorica planetarum.* It should be noted that Finé published a Latin translation of his own book, *Nova Universi Orbis Descriptio* in 1532, just after his appointment at the Collège Royal.[20]

To go back to *La Théorique des Ciels,* we should notice that it was an up-to-date astronomical treatise, and that it offers a good example of how a book could travel between Latin and the vernacular. First, the university book by Peurbach appears in Vienna. Then, Finé gives it a new form, a new presentation, in the vernacular, so that it is appreciated and promoted among cultivated vernacular readers through a vernacular treatise: it is discussed by preceptors and in academies; this makes plausible the introduction of the new presentation at the level of university colleges and then the book is translated into Latin.

But what this new presentation consisted of? These vernacular publications appear as works written in a cultivated style, in humanistic French and in a compact and stylized form. They were usually short and light compared to earlier treatises.

[19]On this genre, see especially (Byrne 2010).

[20]For which scientific curriculum was expected at the time, which new books appeared in the sixteenth-century and for which public, see (Pantin 1995).

They strove for saying the minimum necessary to apprehend a topic, by providing the basic vocabulary and the crucial problems.[21]

However, Finé (1543) published also some books in French concerning the disciplines which did not have a classical tradition or a history of university teaching. In 1543, Finé published *Les Canons et documents très amples, touchants l'usage et practique des communs Almanachz que l'on nomme ephemerides. Briefve et isagogique introduction sur la judiciaire astrologie*, reprinted in 1551, in 1556 and after his death in 1557. Almanacs have their own special history. Present throughout the history of mathematics, they connected astronomers to people of basic level of literacy, university teaching to peasants. In fact, it was among the most well spread genres, stable in spite of its transformations: in the late Middle Ages, these started to include the position of celestial bodies, as opposed to the astronomical tables present in Ptolemy's almanac. So, while the genre of the almanacs belonged in fact to university tradition, its Renaissance version had an existence outside the universities. Notice that it was also one of the genres profiting from the printing press: it is a fact that is made clear by the fact that Gutenberg published the first printed almanac in Mainz in 1457, 8 years before the Bible.

What was the readership for these texts? Immediately we think of people who were not necessarily fluent in Latin, such as the members of the court in general, but also of the court academies (the ones studies by Yates)[22]; we think also of the aristocracy in general, including women and of cultivated merchants and their children, sent to school but not to university. However, there were also students who could appreciate mathematical books in the vernacular.[23] Astrology, for instance, had a vernacular audience and we know that readership of these works included medical students.[24] Furthermore, some of the sixteenth-century 'academies', I mean here the military colleges outside the university where young aristocrats were sent they did not intend to prepare for the university and they devised a curriculum entirely in the vernacular.[25] In this connection, it should be noted that part of the readership for cosmographical books was constituted by women[26]: while mathematics in our restricted sense does not seem to count female professionals at the time, cosmography included some women, at least at the level of calculators and almanacs writers. This aspect is particularly important here because it points to the open question of what kind of consistent education in mathematical sciences was offered to women at the time. Generally speaking, it probably consisted of sharing a preceptor's teaching with male relatives. Notice that preceptors or private teachers should in fact be seen as the most common opportunity for formal education in mathematics in sixteenth-century France, both for male and female pupils. In these

[21] On this point, see for instance (Dubourg-Glatigny and Vérin 2008).

[22] See the classical (Yates 1988).

[23] Eisenstein (1979).

[24] About the vernacular audience for astrological publications see (Schechner 1997).

[25] See the classical study (Chartier et al. 1976).

[26] Pritchard (1990).

teaching situations vernacular training in the "new" mathematical disciplines could be combined with Latin and Greek training for the Classics and classical disciplines.

In 1556 appeared posthumously another vernacular work by Oronce Finé: *La composition et usage du quarré géométrique, par lequel on peut mesurer fidèlement toutes les longueurs, hauteurs et profunditez.* This work belongs to the genre of practical geometry, of the type concerning instruments. While practical geometry in general had a university tradition, this aspect of it had been transmitted outside university teaching. Again, Finé produced a combination of the two traditions, and chose the vernacular to publish it.

In fact, bilingualism was most common status among sixteenth-century intellectuals. But bilingualism of authors took, especially in France, the form of systematic self-translation. Language can often indicate to us which kind of teaching was the target of the book. Latin was meant in general for traditional schools, such as colleges and universities, while French is for abacus schools – but they were rare in France-, and preceptors or private teaching, a very usual institution for mathematical sciences, if we remember that private teaching in mathematics could take the form of apprenticeship at the mathematician's household. Given that the change in public implies also a change in the reading process, what is good for classes (a written encyclopaedic reference book for oral Latin, like Finé's *Prothomathesis*) is not necessarily good for self teaching (mostly vernacular).

8.2.2 Peletier

After Finé's first steps in vernacular scientific publishing, we find again one of his disciples at the Collège Royal in Paris, Jacques Peletier du Mans, as one of the main promoters of the vernacular scientific book project. Peletier began when he was 27, in 1541, by encouraging the use of French in literature through his translation of Horace: the theory of imitation presented therein contributed to the *manifesto* of the *Pléiade*.[27] As he explains in *L'Art Poétique d'Horace, traduit en vers françois*, if the goal is being creative in science, writing in French is the precondition for being creative; the means by which the writing of science in French may be promoted are various, but the most basic and fundamental is the development of a new orthography. In the letter to the reader, he explains that, "just as the word signifies the thought similarly orthography signifies the word."[28]

This approach is clearly presented in the introduction to the *Dialogue de l'Ortografe et Prononciation Françoese* (1550).[29] The *Dialogue* itself represents a debate between Peletier's friends, Jean Martin, Théodore de Bèze, Denis Sauvage

[27]Peletier (1541).

[28]'Tout ainsi que la parole est significative de la pensée, semblablement l'orthographe de la parole'. (Peletier 1541, fol.2).

[29]Peletier (1550).

and Jean Paul Dauron, at the house of the publisher Vascosan.[30] Only two points about the *Dialogue* need to be mentioned here. First of all, Peletier's insistence on the notion that language comes from the people, and is transmitted by contact with the people. Across time and space, only writing can provide a substitute for the normal transmission of language, which is oral. However, writing is more than a mere substitute for speech. In the *Dialogue* he explains that

> This is why <writing> should not so much depend on speech but on understanding, given that the plus which we get from writing is the understanding of sense.[31]

In this way, Peletier states clearly that writing has autonomy with respect to speaking: writing has, in fact, an advantage on speaking, the reader will focus on the meaning. Furthermore, writing does not convey pronunciation, and this is why the study and improvement of orthography is so important as a tool for bridging the gap between written and spoken language.

The second point is at a different level, and concerns more directly the project which Peletier was to pursue throughout his life: the writing of scientific books in French. Peletier writes:

> Our mathematics has never been doing as well as now, nor in a better position to be understood in its perfection. And given that its true is manifest, infallible and constant, think of what immortality it could give to a language if it were written in it according to a good and true method. Let us look even to the Arabs, who in spite of being very far from us and almost in another world, yet some Europeans have wanted to learn their language, especially because of Astrology, and other secret things they have dealt with in their vernacular. This is indeed quite unfortunate for we know what sophistry they have mixed with Medicine and even with Mathematics. Still, they made their language necessary for the study of all this. We should think why we don't do not only as much but without comparison more of our language?

Peletier indicates quite clearly that his plan is to impose French as the language of science or, to paraphrase him, to make that language necessary for the study of science ('rendre notre langue requise en contemplation des sciences'). Peletier took this program seriously.

As we have seen, he had already produced, five years before the publication of the *Dialogue*, his edition (with commentary) of Gemma Frisius' *Arithmeticae practicae methodus facilis* (1545)[32]: from this prestigious beginning in Latin with a work that was meant to be used in teaching at the colleges in France, Peletier moved on to his own mathematical works, which are always marked by his concern for a perfection of the form of presentation and by an explicitly reasoned rhetorical, logical and orthographic strategy.

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[30] The dialogue was actually written in Lyon, after the religious wars and the fragmentation of his circle made Peletier du Mans leave Paris. See the classical (Zemon Davis 1964).

[31] 'Voela commant elle ne doet point ètre tant sugette a la prolation qu'a l'antandemant, vu que le plus que nous retirons de l'Ecritture cét l'intelligance du sans.' (Peletier 1550).

[32] Frisius (1545).

Peletier's program in French became more precise with the four following works: first his own book on arithmetic, *L'Aritmetique departie en quatre liures, à Theodore de Beze* (1549),[33] then *Le dialogue de l'Ortografe* (1550),[34] third, his algebraic work *L'Algèbre* (1554)[35] and lastly *L'Art poétique* (1555).[36] Peletier's publications in French after the beginning in Latin with Frisius' *Arithmetica* underline his own personal transition from colleges where the *Arithmetica* could be used to an aristocratic cultural "*élite*" less familiar with Latin, that is, circles connected to the Court, as well as preceptor's teaching.

After this sketch of the larger programme of publishing vernacular scientific books according to Jacques Peletier du Mans, let us see how he put it to practice in his mathematical works and in what teaching situations they could be used.

8.3 Jacques Peletier's Works in Mathematics. Bilingualism and Self-Teaching

In 1549 Peletier published his own book on arithmetic, *L'Aritmetique*.[37] Here the new language means that after introducing a new topic at the French colleges by his edition of Frisius' book, Peletier introduced it to the court by means of a newly conceived book of arithmetic written in French. Peletier is responsible for many innovations in this and other mathematical texts, starting with devoting separate manuals to restructuring the domains of practical and commercial arithmetic on the one hand and algebra on the other: this was new in France. Through changes in the rhetoric of the manuals Peletier made them more acceptable to the wide audience of the court and to the *noblesse de robe* connected with it. In fact, Peletier's arithmetic contains many more commercial rules than contemporary books, including Frisius', and the four introductions to the four books orient the cultivated reader towards a new view of mathematics in which this newly conceived practical arithmetic (not only astronomical, but containing abacus arithmetic and algebra) is presented as a crucial part of the *quadrivium*.

For Peletier, abacus school's arithmetic and algebra were legitimate fields of speculative knowledge, in fact true sciences. In this sense, aristocrats should consider them positively and even practice them insofar as they represent the beneficial part of commerce. In this he was ahead of his time, since a positive view of commerce and interest was not common before the time of Richelieu. Here it is important to stress that he consciously theorized the features of a scientific book. This, I argue, is an indication of Peletier's increasing interest in the process of

[33]Peletier (1549).
[34]Peletier (1550).
[35]Peletier (1554).
[36]Peletier (1555).
[37]Peletier (1549).

printing and the possibilities of dissemination offered by it. This is confirmed by the fact that, once again, while his works were being printed, Peletier lived at the house of the publisher, in this case the Marnef family in Poitiers, even though the book had been written in Paris. But, more importantly, the Marnef were particularly aware of the ongoing orthographic reform, and ready to be leaders in the field, even competing with the main Parisian publishers of scientific books, such as Cavellat and Wechel. With the *Aritmetique* they became the disseminators of Peletier's orthographical reform. In fact, the *Aritmetique* is the first book that might have satisfied Peletier regarding its fidelity to his orthography.

The four books of *L'Aritmetique* contain a substantial introduction to what can be called Peletier's thought. In fact each of the *prooemia* of the four books justifies the novelty introduced in the book. The first prologue justifies the publication of a newly conceived book and comes before a book on the theory of numbers. Already from the beginning, Peletier detaches himself from the style and also the matter of practical arithmetic university texts, oriented to classification and exhaustiveness. Peletier privileges concision and chooses his priorities. He gives only the indispensable definitions for operations and introduces already at this first level the simplest form of the rule of three. In other words, Peletier simplifies the matter, pursuing in the process initiated by Gemma Frisius. In order to justify these choices his first *prooemium* concerns the principle of authority, the *auctoritates*. Peletier wonders why great authors of a discipline follow each others in a time and place as if they engendered one another, while other times and places, in spite of the efforts of some authors, remain without development and creativity. Peletier mentions the golden age of Greek philosophy and opposes it to the dark centuries where neither Porphyrius nor Boethius succeeded to improve the conditions of civilization. The only exceptions were Ptolemy and Galen, whereas the majority, despite their good will, managed only to produce "barbarousness in letters, sophistry in philosophy, darkness in mathematics, quackery in medicine, hypocrisy everywhere".[38] But the earth untilled for a long time becomes fruitful again:

> Which time has ever been more flourishing in Philosophy, Poetry, Painting, Architecture and new inventions in all things necessary to human life than ours?[39]

This is the moment, for mathematical sciences, to raise again, because everywhere mathematics is on the crest of the wave, it is the time of illustrious men in Germany, Italy, Spain and even France can make a name for itself. It is up to the sovereigns to encourage it, or else risk humankind falling back into hypocrisy. This *prooemium*, under an ordinary appearance, introduces the principle that some contemporary scientists are to be considered on the same footing as the major

[38] Peletier (1549, f.iii^{r}).

[39] "Quel temps s'est-il jamais trouvé plus fleurissant en Philosophie, Poésie, Peinture, Architecture et inventions nouvelles de toutes choses nécessaires à la vie des hommes que le notre?", *L'Aritmetique*, Ibidem.

authorities of the classical past. This shows Peletier's consciousness about the value of new discoveries and also of the "new disciplines", that is, of Arabic mathematics reformulated and acknowledged in vernacular languages of the Latin West.

This attitude seems to depend on Oronce Finé's teaching and practice, Peletier's mathematics teacher. Finé will be explicitly cited later, in the third book, in connection with the mathematically crucial theme of the extraction of roots. The section bears the meaningful title of *About the Use of Roots: where incidentally some excellent Mathematicians of our time are mentioned.*[40]

Finé is mentioned together with algebra authors, after Christoph Rudolff, Michael Stifel, Luca Pacioli "to whom we owe the fact of having enriched us of their labours".[41] Peletier goes on: "Comme en toutes parties des Mathématiques fait très bien son devoir notre renommé Oronce Finé lecteur du Roi en l'Université de Paris". Clearly Peletier is very aware of the mathematical renaissance: it is obvious to him that his commitment to *humanitates* does not imply to neglect present time concerns, discoveries and priorities, nor to subordinate it to a supposed golden age in science. The stake for Peletier is to understand the sequence of contributions by mathematicians in the course of time, contributions which are compatible and cumulative. This is explained in the fourth *prooemium*. While Oronce Finé, in his oration on the importance of mathematics had maintained that without mathematics nothing would exist, neither divine nor human, that mathematics is constituted once and for all in God or together with the Creation, Peletier chose not to found the justification for mathematics on it, and opted to enter immediately into the flesh of human creation of mathematical disciplines.

The constitution of sciences and of scientific traditions is the theme of the fourth *prooemium*, in which Peletier asks the question of why "the practice and use" of arithmetic have not been written by the ancients mathematicians, the Greeks.[42] The answer given by some authors, says Peletier, is that the ancients had avoided to assume what is certainly a burden: to collect so many rules, and rules which belong to the "mechanical people".[43] Some others, Peletier adds, maintain that the Greeks were interested in spiritual things and neglected or ignored practice. To this opinion Peletier opposes a thesis on the origin of demonstrative mathematics:

> I shall never be of these. For it seems to me to be a great mistake to believe that Theorems have been so well presented and in such a good order as we have them from the authors, without finding their certainty and experience by calculation.[44]

[40]"De l'Usage des Racines: là où incidemment est faite mention d'aucuns excellents Mathématiciens de notre temps".folio 003F.

[41]"Auxquels tous nous sommes grandement redevables pour nous avoir enrichis de leurs labeurs".

[42]This question will be taken up again by meaningful authors such as Descartes in connection with Greek geometrical analysis: how is it possible that the Greek did not transmit to posterity the method of analysis if they obtained their results by means of it?

[43]"gens mécaniques".

[44]"Je ne serai jamais d'avec ceux-ci. Car l'erreur me semble grand, de croire que les Théorèmes aient été si bien rédigés et en si bon ordre comme nous les avons, par les auteurs, que premièrerment ils n'en eussent trouvé en ratiocinant la certitude et l'expérience."

The second book introduces fractions and its *prooemium* elaborates on the literary themes of *brevitas* and its costs at the level of clarity. The question is whether one of the two should be preferred to the expenses of the other. This is of interest to us especially because of the reference to printing[45]:

> Among erudite people, my friend De Bèze, has been debated for a long time and is still to be decided which is more profitable to pursuing arts and disciplines: that the teachers, when they write them, deal with them clearly and extensively, or obscurely and shortly. Those who are of the latter party say that difficulty causes that one reflects for a longer time, or reads more attentively what one desires to know: that by reading it and rereading it the readers grasps it with greater certainty, and one formulates oneself the special doubts and lively reasons belonging to the matter, and by this means one comes to the knowledge of the points which are not explained, and acquires a habit and a firm resolution of what ones looks for. This does not happen when things are easy: for human mind being always curious to know, and to go on further, after having listened a passage, throws itself on the other, without taking the time to engrave <to print> nor to incorporate into memory what one needs to remember. And they say that since printing has been invented one has not seen so many people of learning or so solid as it was done in the past, because since people have a quantity of books at leisure, they want to embrace not only many authors of one profession, but also many different professions. By charging their mind of so many things, this is the cause of the fact that they are forced to let go a great part of each on the road, and in fact they are not satisfied of any.[46]

I have commented this passage elsewhere,[47] given its relevance for the definition of the new topic, algebra, by its two main features: brevity and clearness. Let us just notice here that Peletier does not say himself that printing should be considered a real risk for the avid reader, but rather that this is what some contemporaries maintain. They are those who think that easiness and availability of texts are a paradox: instead of providing "food for thought" they would produce passivity and superficiality in learning. On the opposite, difficult texts would be a good thing, because they force to make a voluntary effort. Peletier insists that clarity, together with brevity, are the tasks of the true author. But it is interesting for us to see that printing was not necessarily seen as a progress, and that spreading of information was seen also as an incentive to superficiality. Peletier show here to be particularly conscious of the challenges and the risks of the various forms of communication, as in the *L'Art Poetique* he shows to be conscious of the challenges and risks of the various styles of communication. Quantitative increase of the access to information does not by itself insure an increase of knowledge. This point is fundamental for us, insofar as a teacher plays the role of a guide in the universe of culture and a guide who makes sure that knowledge is actually acquired.[48] In fact, we see appearing here a reflection on the practice of self-teaching, as printing was connected to such a degree of availability of books that the chain of transmission through teachers as a

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[45]This can be read as a dialectical question, as I have done in (Cifoletti 2001).

[46]*Ibid.,* folio 27v.

[47]Cifoletti (2004).

[48]On the problem of mastering information and having guides through the universe of culture, see (Blair 2010).

guarantee of a good interpretation could not be assured anymore. Obviously this has to do with the Reformation and its trust for the reader of Scriptures. Now also the reader of mathematical texts should be trusted to understand and master the matter.

Also about fractions Peletier applies his theories to his mathematical text. After having maintained that to combine clarity and brevity is possible, Peletier authorizes himself to reduce even more drastically the theory of decimal and astronomical fractions. This is all the more noticeable, given that astronomical fractions is precisely the field of interest for Peletier, in which he distinguished himself with his *Annotationes* to Gemma Frisius' *Arithmeticae practicae methodus facilis.* As we have seen above, his *Annotationes* on this topic were particularly appreciated in the sixteenth-century.

The third book is about roots extraction and proportions, both fundamental operations in astronomy and commercial arithmetic. It is certainly a great achievement among mathematical books of the time. Peletier introduces the book by the discussion of vocabulary and apologizes for his Hellenisms. He does not hesitate to simplify a great amount of matter: he selects the two themes and exemplifies them thanks to his experience with Gemma Frisius's *Arithmetica*, where both roots extraction and proportions are developed. Faced with a typically Euclidean topic, Peletier had an enormous tradition before him. He chose to rely on his teacher Oronce Finé, and precisely on his *Protomathesis.* The most evident change is obviously the brevity of Peletier's text with respect to his predecessor. Peletier also achieved the clarity his *prooemium* to the second book had promised.

Finally, the fourth book is the one which goes deeper into commercial arithmetic, again resting on some aspects of Gemma Frisius' results, but also independently. All commercial rules are explained, not only the various cases of the rule of three, but also the rule of false position and double false position. Its *prooemium* speculates on the missing written tradition of practical (in the sense of commercial) arithmetic. Peletier speculates on the connection between the rule of false and the rule of three:

> The Rule of False, which the Arabs call rule Catain is called in this way because from a false supposed case it teaches how to find the true. And among all the vulgar rules it is the one which is more beautifully and largely used. It has two parts, one of a single false Position, the other of two. The Rule of False of one position operates almost in the same way as the rule of Three, except that in the rule of Three we have three known terms: here we only have one (I mean, entering in the operation) in analogy of which we form the other two, one by multiplying, the other by dividing.[49]

After the treatment of the rule of double false position, Peletier stresses the role of Gemma Frisius's combination of the rule of false and the rule of three and in fact

[49] *Ibid.* folio 86: "La Regle de Faux, que les Arabes appellent la Regle Catain, est ainsi ditte, parce que d'un cas faux presupposé, elle enseigne à trouver le vrai. Et est celle de toutes les Regles vulgaires, de laquelle l'usage est plus beau et plus ample. Elle a deux parties, l'une d'une seule Position fausse: l'autre de deux. La Regle de Faux d'une position a presque pareille operation a celle de la Regle de Trois, excette qu'en la regle de Trois nous avons trois termes cognuz: ici nous n'en avons aucun (i'entens qui viene en operation) à la semblance duquel nous en formons deux autres, l'un multipliant et l'autre divisant."

the rule of algebra as will be presented by Stifel, explicitly mentioned by him: "On the amplification of the rule of false through the extraction of roots, very ingenious invention of Gemma Frisius".[50]

History of inventors, brevity and clarity, theory and practice, mathematics and language, writing of practical arts: the *prooemia* are very cultured presentations. The ideas are clearly based on ancient Greek philosophy and could introduce to a university text traditionally in Latin. Instead, they are the philosophical justification of a mathematical book of a new sort.

Peletier realized his programme by publishing several mathematical works. We shall look more closely at the ones he chose to translate. *L'Algebre,* printed in 1554,[51] is among the mathematical texts by Peletier the richest in philosophical remarks on the role of the mathematical author, the importance of language, and about algebra as the art of thinking in mathematics.[52]

What matters here is that Peletier chose to start with a French algebra book, importing in the French vernacular the mathematical contents already expressed in Latin by Stifel and Cardan. The book is written in the elegant style of the vernacular scientific books we described earlier: short, clear, with classical references and some philosophy. In 1560 Peletier published a Latin version of his algebra.[53] How should we interpret this fact? We can at first notice that this version of the book has been used at the level of university colleges, at least at the Collège Royal, as suggested by the annotated copy belonging to the royal professor Henri de Monantheuil. There are no similar indications concerning the French edition. Other hints connecting this work to its use in colleges are in the very content of this version of the book. For, there are important differences between the French and the Latin Algebra. First of all, all the passages concerning the linguistic reform and the use of algebra as an art of thinking are omitted. This is compensated by an important dedicatory letter, containing a critical analysis of the current mentality, with special reference to its lack of honesty and understanding of the past. A plausible conjecture is that in the colleges the mathematical content should be clearly distinguished from other, extra-curricular concerns. This would also avoid disagreements between the author and the teachers which did not bear on the actual mathematical statements. The two books differed also on other points: in the French version the structure and most of the content is borrowed from Stifel's *Arithmetica Integra,* whereas the Latin version includes a concern for demonstration of the rules of solution, and in this connection Peletier gives references to the relevant propositions in Euclid's *Elements.* The importance of proofs and the references to classical author are indeed typical features of college textbooks of the time, in all mathematical disciplines.

Jacques Peletier du Mans published another mathematical book in Latin and French: this is the *De Usu Geometriae* (1572) and its French translation *De l'Usage*

[50]"De l'amplification de la regle de Faux, qui est par extraction des racines, invention tres ingenieuse de Gemme Frisien".

[51]Peletier (1554).

[52]I have written on these topics elsewhere: (Cifoletti 2006a).

[53]Peletier (1560).

de la Géométrie (1573). This text is emblematic of Peletier's style in mathematical books: short, clear, in small format (in 8°), with few engravings, in his typical, ennobled French. Peletier published this book while he was employed by the Duke Emmanuel Philibert of Savoy as a preceptor of his son Charles Emmanuel. Peletier had served Emmanuel Philibert since 1570 and participated in his reform of the army and the public administration in Piedmont and Savoy. It is probable that French had been the language of Peletier's private lessons to Charles Emmanuel. In this case it is likely that a canvass of the French text was ready while the Latin text was printed. The dedicatory letters give a hint of Peletier's plans. The first, to Marguerite of Valois and her husband Emmanuel Philibert indicates that the purpose of the print edition in Latin was to give this text a widespread use in educational institutions, such as those founded by Emmanuel Philibert. The dedicatory letter of the French edition is to another eminent nobleman interested in military reform, that is Albert de Gondi, comte de Retz, *gouverneur et lieutenant general pour sa majesté*. The purpose of the French edition was to pursue the vernacular scientific book project of Peletier's youth and to open Peletier his return to Paris after the years in Lyon and Savoy.

Finally, we can recall another important publication by Peletier: his version of the first six books of Euclid's *Elements*. It appeared first in Latin, in 1557, and then translated by Jean II de Tournes at the beginning of the seventeenth-century.[54] This format, up to book VI, was typical of college teaching of Euclid. It is not a surprise, therefore, to find it in Latin at first, while by the beginning of the seventeenth-century it is possible to find other instances of teaching of Euclid in the vernacular, in the military "academies" for young aristocrats. In conclusion, we see Jacques Peletier as a crucial figure in the making of a mathematical literature both in Latin and in French. The use of language is consistent with the choice of public and, aside from the instance of practical geometry, so is the managing of the matter in the books.

8.4 Nunes' *Libro de Algebra*: In Connection with the French Algebraic Tradition[55]

Another sixteenth-century cosmographer was engaged in the vernacular scientific book project: the Portuguese Pedro Nunes. Here we shall deal only with The *Libro*

[54] Peletier (1557, 1611).

[55] My research on Pedro Nunes started with the study of Guillaume Gosselin in 1986. I have expressed various aspects of my reading of Nunes' algebra in three talks: "Gosselin's use and critique of Nunes' Libro de Algebra", conference *"Iberia in the Golden Age"*, 31/3-3/4 1993, Imperial College, London, in "*Subtilissima arte*. Pedro Nunes et l'algèbre de la Renaissance", conference *Pedro Nunes and the Science of His Time* 8/11-10/11 2002, U. of Coimbra, as well as in a talk with the same title in Paris, at the seminar of the Centre d'histoire des sciences et des philosophies arabes et médiévales (Paris-Villejuif), mars 2003. My review of Martyn's book on an algebraic ms attributed to Nunes appeared in (Cifoletti 2003b).

de Algebra en Arithmetica y Geometria (1567),[56] which has been an important book in its time. Again, we see a book in the vernacular devoted to a topic coming from the abacus tradition: against the wide Latin production by Nunes, the other two vernacular works concern navigation. The book has retained the attention of the historians of mathematics only up to a certain point before its recent revival connected with the edition of Nunes's Complete Works.[57] Nunes' book does not contain either absolute novelties, as H. Bosmans noticed in his two articles already at the beginning of the twentieth century.[58] He also suggested that the value of the text lies rather in the generality of his approach. According to Bosmans, the most important aspect of this generality, capable of transforming the discipline of algebra, consisted in translating a vast collection of merchant's problems -which constituted a traditional corpus at least since Fibonacci' *Liber Abaci*- in a set of problems on numbers. Aside from this achievement, Nunes is credited of some solutions adopted by later algebraists, such as the determination of the common divisor of polynomials. This justifies at least in part the enthusiasm expressed by mathematicians of the time. Even before publication of the actual book in Antwerp in 1567, Pedro Nunes's algebra circulated as a manuscript, and was appreciated. In France, Jacques Peletier du Mans mentions the book as an important algebra which he has not read yet, and this already in his *L'Algèbre* of 1554 and *In Euclidis Elementa* of 1557.[59] After its publication, the book was rapidly used as a reference. Certainly, part of the reason for this was Nunes's celebrity as royal cosmographer and as an author of important cosmographical works. However, it is also clear that this book responded to a need. The question is to know which need justifies the success of this book, as in the case of Frisius' book. For instance, we have also some contemporary explicit references and commentaries about the *Libro de Algebra*. On this basis, I claim that what the French algebraists were interested in, and in fact most contemporaries, was a text of algebra in which algebra could acquire or *recover,* as they put it*,* its role as an art of thinking, as the art of discovery that the ancients had hidden in their brilliant mathematical constructions.

As I have shown[60] elsewhere, mathematicians had a larger public across Europe, namely the readers of arithmetic and algebra in the sixteenth-century, who often had at least some level of a humanistic culture in the vernacular and at best a juridical, medical or theological training. For such a humanist public, mathematics was the form of thought closest to dialectic. Nunes wanted to recover algebra's original "ordre et méthode" which made it an art of thinking. The structure he tried to give to the art of algebra derived from his interpretation of Euclid, and in fact he ascribed algebra to Euclid. This attitude came from the humanist idea that considered every

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[56]Nunez (1567).

[57]See (Leitao 2010), in particular Vol. VI. *Libro de algebra en arithmetica y Geometria*, which gives a view on the recent literature on the topic.

[58]Bosmans (1907–1908, 1908).

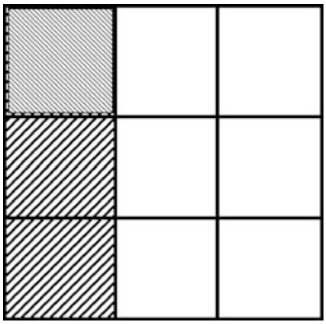
[59]Peletier (1554, 1557).

[60]Cifoletti (2003a).

discovery or invention, in mathematics as well as in poetry, as founded on the imitation of the great classics, on a translation and understanding of their works.[61] It is up to us, now, to reconstruct the reconstruction of the classics proposed by Nunes, because it is at this level that his work was considered profoundly innovative by sixteenth-century readers.[62] Nunes, more than any of his contemporaries, takes up the challenge to provide justifications for the solution formulas of first and second degree equations. Nunes calls this part *doctrina* and puts it at the foundation of the very subtle art of algebra. This is the way to see why this book has been considered pedagogically successful, and for this we need to enter a little into his mathematics.

Nunes follows Pacioli's for the names of the 'powers of the unknown': *Numero* is called a quantity meant as composed of units, either a integer, or a fraction, or a root, even when it is surd [in our terminology: irrational]. *Cosa* is called the root of any square. *Censo* is called the square which is born from this root.

Nunes reminds us that, according to Aristotle, we get the number by the division of continuum and that number is a collection of units. We must therefore imagine to obtain number and units in lines, units in surfaces, units in bodies, by dividing them into parts. We could divide in equal or unequal parts, Nunes says, but mathematicians divide in equal parts in order to establish relations between the parts and the whole, which are also represented by a number. Mathematicians have the habit to divide the plane surface in equal and square parts, any of which is called unit, in conformity to the linear unit it has as side. In order to calculate the area and the volume we establish that the value of the figure is represented by the product of linear units contained, for instance, by the two sides of the triangle. By root or cosa we should understand the rectangle which has the side of the square as a basis and the unit as height. Nunes claims that in any root of the square there are as many square units that there are roots constituting the square. (As in the figure: Here the strip of three vertical shadowed squares constitutes one root.)



[61] Morse (1982).

[62] See (Cifoletti 2006a, b, c).

The number of square units contained in the square is therefore the value of the figure, whereas the value of the unknown is the number of linear units contained in the side of the square. As a consequence, we determine the solution of the equation, that is the value of the unknown, thanks to the equality posited, by counting the number of roots.

Nunes concludes by saying that many are mistaken by conceiving the root as a segment, and do not think that if the root was a straight line it could not be put as equal to a square.

Obviously Nunes is dealing here with the question of the homogeneity of the terms, but also of the correspondence, much more powerful than a simple illustration, that he wants to establish between geometrical quantities and monomials.

The reader is probably surprised by Nunes's statement, given our habit to see the root of a square precisely as a line, thanks to the introduction of the unit in the famous page of the *Géométrie* by Descartes. However, Nunes "intuitive" interpretation works.

Nunes states two rules:

1. There are as many units in any root, that there are roots in the square
2. The root of the square is a surface, but the side of the square is a line

Thus, the Root is different from the value of the unknown because Root is a geometrical entity, the stripe, while the value of the unknown is a number.

Note that if x^2 is censo, b the coefficient of the unknown is also number of units in the cosa, while x is also the number of the cosas (rule: number of units in cosa = number of cosa in censo).

In this framework, Nunes gives a proof a case of the solution formula for the second degree equation, based on the main references to Euclid.[63] What Nunes adds is the use of the *doctrina*, which establishes the correspondence between the two entities having a different geometrical meaning, but the same number: the number of square units in the root and the number of roots in the square. The value of the unknown is such number.

Nunes has attempted, with success, to advance in the geometrical interpretation of algebraic procedures. More than any author, Nunes succeeds to make explicit the sense of the algebraic operations. The geometrical sense of elementary operations has a heuristic role, but also what we would call a foundational role: at stake is the transition from the *ars rei et census* as it was presented in the abacus schools, to an art. Nunes stresses also that geometric progression, that is continuous proportion and the constant ratio between powers of the unknowns are the foundation of algebra. Nunes claims to have uncovered that *mathesis universalis* which is prior to arithmetic as well as to geometry, and that analytical doctrine which Euclid and Archimedes have used and hinted to without revealing it. Furthermore, that this art is in fact an art prior to arithmetic and to geometry guarantees that is has its place in the quadrivium but also that it can be legitimately used beyond practical calculation.

[63] I examine this point in detail, and compare it with Pacioli's treatment, in (Cifoletti forthcoming).

The reason why to start the book with the demonstrations of the solution formulas, then to structure it as an *algorismus,* and finally to get back to the theory of equations becomes clear: Nunes wanted to give algebra the foundations in the *doctrina*.

He actually gives a hint to this in the dedicatory letter to the King's son Henrique: the *doctrina de ciencia especulativa* is put at the very beginning because "a lot of time is usually spent in it, while in this way it can be learned in little time, easily and without the help of a teacher."[64]

In this sense, the theoretical summary at the beginning is especially important for some readers, that is for self-taught, and less for others, who could have a regular training in elementary arithmetic and geometry. Nunes should be acknowledged as the mathematician who actually connected directly the algebraic powers of the unknown to the notions given by Campanus and Zamberti in their interpretation of the arithmetical books of Euclid's *Elements* (founded on Theon of Alexandria) as well as to Jordanus' *Arithmetica* statements, in order to give foundation to algebra. Campanus and Zamberti had read the arithmetical books of the *Elements* (VII: VIII and IX) by associating them to the II and VI. This reading of Euclid became common, but few actually made this foundation explicit as Nunes did. This way is based on an interpretation of Euclid, each author proposing his own. Pacioli and Tartaglia had even provided a version of Euclid's *Elements*: Pacioli in Latin in 1505, based on Campanus's edition against the recent edition by Zamberti, whereas Tartaglia translated the *Elements* into Italian taking as basis both Campanus's and Zamberti's edition.

The dispute on Euclid was about main philological points aiming to establish the text of the *Elements*[65] but was also strongly philosophical insofar as it determined the interpretation of some fundamental aspects of the text: the connection between arithmetical and geometrical books, and most particularly the role of "geometric algebra", that is the theory presented in the books II and VI.[66] Pacioli's and Zamberti's dispute was overcome by the Lefèvre d'Etaples's edition (1516) and the Greek editio princeps together with Proclus's commentary (1533).

Nunes seems to echo what Tartaglia wrote in the frontispiece of his "Euclid"[67] promising:

> An extended presentation added again by the translator himself, so clear that any mediocre mind, without any notion oor help from any other science will be easily capable of understanding it.

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[64]Nunez (1567, folio 3).

[65]The story of the contest between Pacioli and Zamberti has been described by Rose (1975) and more recently by Goulding (2010).

[66]I have sketched some of the philosophico-mathematical stakes of this debate in Cifoletti (2006c).

[67]"Con un'ampla espositione dello stesso tradottore di novo aggionta. Talmente chiara, che ogni mediocre ingegno, senza la notitia, over suffragio di alcun'altra scientia, con facilità sera capace a poterlo intendere." Tartaglia 1543.

8.4.1 The Dedicatory Letter

The dedicatory letter of the *Libro de Algebra* to his pupil the Infante Henrique, dated 1564, contains several indications as to Nunes' motivations and agenda and about the way he saw his work in the context of his time. It is the section in which Nunes gives more details about the social purpose of his work. Here I shall mention the themes we have already discussed for other authors, such as the relationship between theory and practice, the use of the vernacular in science (notice that the letter is in Portuguese, while the book is in Spanish), self-taught readership as well as Nunes' sources and the origins of algebra. Nunes writes:

> Algebra is an easy and short calculation to know the unknown quantity, in any problem of Arithmetic or Geometry, and in any other art making use of calculation and measure, such as Cosmography, Astrology, Architecture and Mercantile arts *(Mercantil)*[68]

Nunes states the possibility of using algebra in many mathematical sciences or rather in all those which are about calculation and measure: not only arithmetic and geometry, but also cosmography, astrology/astronomy,[69] architecture and the mercantile arts [Mercantil]. The potential universality of the use of algebra is a commonplace of all algebraic books of the time and Nunes includes, in fact, the mathematical sciences that he practices. Furthermore, Nunes adds here what can be considered an indication of the social role of algebraic books. He observes that in Italy there are many abacus schools, and in all towns there are teachers of counting in arithmetic and geometry who draw a salary form the town itself. But now also in the city of Lisbon mathematics is so wonderfully used in commerce and already financed by the King, given that 40 *contadores* are have a salary at the royal *fazenda* (*sua fazenda*). Therefore, the time has come, Nunes declares, that also in the city of Lisbon a book should appear without presupposing a doctrine of the speculative science, but so as to make it possible to learn it in little time, easily and without the need of a teacher.

Concerning the question of the use of the vernacular in science, he claims that he had written the majority of the text in Portuguese 30 years earlier. Jacques Peletier had already published this information in 1554 and in 1557.[70] However, this new version is, that the rest of the book is in Spanish in order to reach a wider public and because there are almost no books in Spanish devoted to Algebra.[71] He also mentions the conditions in which the book can be received: Nunes' remarks are very important as a witness of the situation of abacus schools discussed in the first chapter. It is evident from what Nunes says that in Portugal the abacus schools were not as present as they were in Italy. However there where 40 calculators paid to

[68]Nunez (1567, f.1).

[69]At the time astrology was computational astronomy and our astrology, whereas astronomy was natural philosophy, this is why I gave both terms in my paraphrasis: Nunes has *Astrologia*.

[70]In *L'Algèbre* and in *In Euclidis Elementa:* (Peletier 1554, 1557).

[71]Although, as to the presence of algebraic books in Spanish, it is surprising that Nunes does not take into account Marc Aurel *Libro primero de Arithmetica Algebratica.*

practise their art the court, which is a large number, and a new and growing public in the readers of vernacular books who had a sufficient general culture to learn a discipline without a master and wished to learn algebra outside the abacus schools. It was precisely this public who, according to Nunes, wanted the *doctrina speculativa* and convincing demonstrations in a compact form. Notice that Pacioli, one of the few authors explicitly mentioned and praised by Nunes, had already tried to found algebra on Euclid's *Elements*: in fact, he called *practica speculativa* that part of his treatise dealing with what we would call algebra and the demonstration of the solutions. Nunes sees this as a way to uncover the hidden practice of mathematics which Euclid and Archimedes presuppose.

Later we find a remarkable passage:

> How much better it would be if the Authors who wrote in the mathematical sciences had left their inventions written in the same way and with the same reasoning that they had used to get to them. And not, as Aristotle said in his *Mechanics* about artisans, who show only the external part of their machines and hide the *artificium* in order to be considered praiseworthy. In any art invention is very different from tradition and we should not think that all these propositions by Euclid and Archimedes were obtained in the same way that they used to transmit them.[72]

This explains the initial passage of the dedicatory letter, in which Nunes states that algebra is necessary for the practice of some books of Euclid and of Archimedes: for, here he says that algebra is the invention, whereas those books give us the tradition,[73] algebra gives the royal road or the *mathesis universalis* premises of arithmetic and geometry. Here Nunes deals with the *topos* of the hidden analysis of the Greeks, known from Peletier, Viète and Descartes among others: Nunes, not without irony, represents the great Greek mathematicians as highly specialised artisans, hiding their method of discovery in order to make their results look miraculous. This discussion on theory and practice could be incarnated in the contemporary time with the role of abacus schools versus humanistic college and university teaching of algebra.

8.4.2 Motivation for the Foundations of Algebra: Nunes' Sources

The background for Nunes' pedagogical foundations for algebra are his sources. As to contemporary authors, Nunes mentions from the beginning Luca Pacioli.[74]

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[72] "O quan bueno fuera, si los Autores que escrivieron en las sciencias Mathematicas, nos dexaran escriptos los sus inventos por la misma via, y con los mismos discursos que hizieron hasta que pararon en ellos. Y no como Aristoteles dize en la Mechanica de los artifices, que nos muestran de la machina que tienen hecha lo de fuera, y esconden el artificio, por parescer admirabiles. Es la invencion muy differente de la tradicion en qualquier arte, ny penseis que aquellas tantas proposiciones de Euclides, y de Archimedes, furon todas halladas por la misma via que nos las han traido." (*Ibid*, f. 114).

[73] A different thesis is (Høyrup 1998).

[74] Henrique Leitao has developed the study of Nunes' sources, see (Leitao 2002, 31–58)

He says that Pacioli has published the first book on algebra. It is clearly the main source for Nunes in writing his own book. At the folio 323v we find the address of *The Author of this book to the readers*, which begins:

> I thought it a good thing to talk about the books of Algebra which arrived in Spain so far, so that you would read them with judgement and choose the most useful.[75]

In this connection Nunes develops also his criticism of Pacioli, explaining that Pacioli often uses the art without mentioning it, and when he does mention it he does so without getting to the point. This underlines once more that Nunes purpose in presenting the solutions of equations and their proofs at the very beginning of his treatise was to give a short and understandable foundation. A little later he repeats that Pacioli puts the conclusion at the beginning, which is not good for beginners. It is because of this great lack of pedagogy that Nunes has started to write the book and communicated it to some people.

Nunes also mentions a "summa" by Cardano, in emulation of Pacioli: here we should think of Cardano *Practica Arithmeticae* (1539). He comments that this book follows a certain order but it is not homogeneous. Furthermore, he writes, Cardano later edited a book on algebra – here we should think of the *Ars Magna* (1545) – which was completely without order (*que es un chaos*). Tartaglia's *Quesiti et Inventioni diverse* (1554) as well as his *General Trattato* (1557–60) are mentioned and praised for order, clarity and style. However, Nunes remarks:

> But it is not a selfcontained work [obra absoluta], because it refers the readers to to to others books of his, and presupposes some rules which he has not demonstrated and are not in the books of Euclid, from which all this doctrine comes.[76]

This is the clearest indication of Nunes purpose for his choice of starting his book with the doctrina.

In the course of the book Nunes mentions also Oronce Finé's *Arithmetica,* (by which he could have meant the 1532 *Protomathesis'* version or in the independent Parisian edition of 1542) and Jacques Peletier's *In Euclidis Elementa* (1557).[77] I have already recalled that Nunes does not mention Stifel's *Arithmetica Integra* 1544 and it is also noteworthy that Peletier *L'Algèbre* (1554) is not mentioned either.

He also mentions a few classical authors, and it is important to keep in mind that the editions of the classics occupied many mathematicians of the time and had a great influence on mathematical styles. First of all, of course, Euclid's *Elements*, for which Nunes most probably[78] had access to two editions: the first, based on Campanus' version, published by E. Ratdolt in Venice (1482), Zamberti's edition

[75] Nunez (1567, f. 323) I wish to thank Samuel Gessner for several discussions on Nunes' text and particularly on this section.

[76] Nunez (1567, f. 324).

[77] Peletier (1557).

[78] This is also the opinion of the editors of the volume (Leitao and Martins 2002).

of 1505 and also the edition including both Campanus and Zamberti's versions, published in Paris by Henri Etienne in 1516. Nunes explicitly mentions Archimedes, which he probably had access to in the Basel bilingual edition of 1544. The third mathematical authority mentioned is Jordanus Nemorarius, the thirteenth-century mathematician, for his *Arithmetica*. The edition Nunes could have access to is the 1514 Paris edition, edited by Jacques Lefèvre d'Etaples, the great humanist theologian who will edit two years later the edition of the *Elements*.

It goes without saying that there are many other contemporary sources for Nunes' work, even in the rather restricted circle of French humanistic mathematicians. Let us recall at least Elie Vinet, who was probably responsible for the contact between Peletier and Nunes.

Nunes also mentions some texts that are not exclusively mathematical, belonging to a larger humanist culture. It is important to keep them in mind not so much to show that he mastered such a culture, which is obvious, but to see how he considered them as pertinent sources in a book of algebra. The first is Marsilio Ficino's commentary on the Timaeus: we don't know whether Nunes had the original Florentine edition of the *Opera* (1491), the Venitian (1516) edition or the most recent and best selling edition in Basel (1561). The second is an encyclopaedia, Giorgio Valla's *De expetendis et fugiendis rebus*, edited in Venice in 1501. Ficino and Valla are the early masters of that neo-Platonic interpretation of Euclid which will take its final shape with Grynaeus' edition of Euclid's *Elements* and Proclus' Commentary.

In fact, the reading of Euclid's book II in terms of general algebraic quantity as proposed by Nunes seems be rooted in a neo-platonic and most particularly Proclus' reading. His *mathesis unversalis,* whose axioms are common to arithmetic and geometry, is, according to Nunes, nothing but algebra. In the use of Euclid's *Elements* in the book, Nunes' approach follows Campanus', but also Theon's reading proposed by Zamberti, with the goal of finding the correspondence between the theorems on segments of book II of the *Elements* and the theorems on numbers of books VII, VIII and IX. At the same time, Nunes could see, together with the French algebraists, the strong connection between this interpretation of Euclid and the work of Jacques Lefèvre d'Etaples and his programme on scientific manuscripts of late antiquity and with the related reform of logic.[79] Finally, Nunes could take advantage of Euclid's *Elements* in Grynaeus' editio princeps, which includes Proclus' commentary to the first book and the definition of *mathesis universalis*.

This had been already conveyed by Pacioli. But he lacked pedagogy. Therefore Nunes had been motivated to write. In fact, the reference to Pacioli is evident already in the title of Nunes' book. But also the title changes the balance of mathematical disciplines: Nunes wants to explain the meaning of algebra in arithmetic and in geometry, because he thought of algebra as a foundation to both, in the way of Proclus' *mathesis universalis*.

[79] See (Cifoletti 2006c).

8.5 Conclusion and Open Questions

A reference to teaching context is claimed by most sixteenth-century books of mathematical sciences. We have looked at some examples of texts, mostly belonging to the corpus of algebraic books of Renaissance Europe, and questioned the claim. For, it raises the question: which teaching and in what situation?

Teaching presupposes previous knowledge, wrote Aristotle and repeated Renaissance thinkers. But it presupposes also something new to be learned. The new could consist also, paradoxically, in the import of a classical text in the humanistic high culture: this could then imply a translation from Greek into Latin or a new better established Latin edition or even a translation from Latin into a vernacular language. An example of this is for instance the editions of Euclid's *Elements*, the various versions in Latin, culminating in Tartaglia's translation into Italian. A traditional and quite interesting way of answering the question would have been to study how sixteenth-century authors came to see ancient sources and interpreted them in or out a teaching context. The relation to ancient sources is the program underlying the Renaissance, there is a lot to be explored in this direction: but I hope the examples mentioned here can show to what extent sixteenth-century concerns and practices could influence the understanding of the past and of past sources. With this paper I insisted on another way to look at the connection between texts and teaching contexts in the Renaissance, that is looking at the vernacular scientific book project, bilingualism and self-translation as meaningful printing choices. For, another kind of "new knowledge" could consist of the import of a topic coming from abacus schools, when it reached the level of college or university teaching. This is the case of the various mathematical arts promoted by royal cosmographers such as Gemma Frisius and Oronce Fine. This is also the case of the algebras printed in the sixteenth-century, produced mostly by specialists trained at universities and working at universities. To what extent the content of these books was actually transmitted and learned by students is not easy to establish. However, the examples we have seen show an intentional effort to simplify the matter and to practice bilingualism. The authors we have seen are particularly careful about producing books which do not discourage the reader because of mathematical complexity: all the texts mentioned here were considered as self contained, not needing technical pre-requisites. The philosophical justification of the promotion of the vernacular was that creation could occur only in the mother tongue. While creativity could not be guaranteed, certainly the possibility of reading a book in both languages or even only in the vernacular led the reader to being autonomous, was the premise for successful self-teaching. Nunes follows this young printing tradition, but to the conscious pedagogical effort he adds his strong standpoint on cognition: mathematical knowledge was founded, and should be presented as founded, on algebra.

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