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THE ART OF THINKING MATHEMATICALLY

Edited by Giovanna Cifoletti

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MATHEMATICS AND RHETORIC. INTRODUCTION

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> Iam primum ordo est geometriae necessarius: nonne et eloquentiae? Ex prioribus geometria probat insequentia et certis incerta; nonne id in dicendo facimus? Quid? illa propositarum quaestionum conclusio non fere tota constat syllogismis? ...Verum et orator, etiamsi raro, non tamen numquam probabit dialectice. Nam et syllogismis, si res poscet, utetur, et certe enthymemate, qui rhetoricus est syllogismus. Denique probationum quae sunt potentissimae grammicae apodixis vulgo dicuntur: quid autem magis oratio quam probationem petit? (Quintilian, *Institutiones oratoriae*, I, 37)

Rhetoric and Mathematical Innovation

This collection originated in a series of workshops on *Mathematics and Rhetoric* I organised during the academic years 2001-2003 at the École des Hautes Études en Sciences Sociales, Paris. Our purpose was to consider two questions. The first, which had come to my mind in 1985 at the time of my dissertation project, is motivated by the oddity of a genre of mathematical texts that is characteristic of the sixteenth century. In accordance with the mathematical instruction since the late Middle Ages, earlier mathematical texts had fallen fairly neatly into two groups: there were, on the one hand, the classical versions of Euclid's *Elements* and Boethius' *Arithmetic*, brought up to date in that they used the Arabic number system and thereby encouraged its general adoption; on the other hand, there was the material of the abacus schools, which included calculations, numerical problems, algebra and some commercial arithmetic. However, by the mid-sixteenth century, this distinction had blurred, and there began to appear texts of a very different sort. Unlike earlier mathematical texts, this new type dealt with abacus mathematics, including algebra *together* with the classical topics. This material was furthermore presented together with a philosophical introduction. This new genre has tended to be viewed by historians as a typical symptom of the poor state of science in the sixteenth century: Copernicus, Vesalius, Cardano and then Viète appeared as isolated characters in a century full of rhetorical virtuosity and confusion. Given the

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state of mathematics in the century, no wonder if, in particular, there had been no algebraic tradition in France before Viète. The standard story concluded that the rhetorical character of early modern mathematical texts should be taken to suggest that their authors lacked mathematical innovation and therefore had focussed on pure instruction. Given the nature of these texts, it is easy to assume that if their sixteenth-century authors had possessed any innovative mathematics, they would not have camouflaged their innovations in such rhetorical garb. It was against this background that my first question arose: Are these rhetorical texts indeed lacking in mathematical innovation, or does their rhetorical character rather constitute a new mode of presenting innovative mathematics as well as an important logical change in presentation? In short, is there innovative mathematics hidden within its rhetorical presentation? Having reached, through the years, a positive answer to this question, my intent was to ask my colleagues to apply a similar question to their own area of research.[1]

The second question developed from the first, but applied to the history of mathematics more generally. Once I had established that the rhetorical character of many sixteenth-century texts was in fact a deliberate choice of how to organise the new material in terms of an older tradition, the idea suggested itself that rhetoric might have played an important role in the very creation of modern mathematics. Hence, our second question concerned the relation between the rhetorical character of these texts and what early modern thinkers sometimes called 'the art of thinking'. The art of thinking included, roughly, a wide range of methodological strategies whose goal was the acquisition of knowledge and the production of valid arguments. These strategies include Cicero's and Quintilian's oratory and rhetoric (invention, disposition, elocution), but also logic and its early modern version, Valla's dialectic. I had come to the conclusion that the rhetorical character of the sixteenth-century new genre of mathematical texts should be seen as a paradigm of this 'art of thinking'. The Paris workshops offered an opportunity to invite colleagues to work on just this question, of whether or not there

[1] See G. Cifoletti, *Mathematics and Rhetoric. Jacques Peletier, Guillaume Gosselin and the Making of the French Algebraic Tradition* (PhD thesis, Princeton University, Ann Arbor, 1992), also available in *Proquest*.

was such a strong connection between an art of thinking and mathematics in later centuries, and also of whether or not the structuring role of rhetoric in mathematics had existed before or after the sixteenth-century. In the end, I was asking my colleagues, as historians of mathematics or philosophy, to evaluate differently the origins and development of the discipline itself, as well as the social and intellectual circumstances of its making. But our second question also applies to historians more generally, for it implies a need to rethink the constitution of the disciplines beyond the division into *trivium* and *quadrivium*.

Of course, our two questions are closely related, and their answers led to exciting new questions. When early modern mathematicians chose not to put their innovations front and centre, did they have philosophical reasons –methodological or epistemological—for their mode of presentation? Once we had decided that the rhetorical character of these early modern mathematical texts constituted itself a mode of the art of thinking, then it seemed clear that these texts must in fact contain both mathematical and methodological innovations, and moreover that these innovations had important philosophical and scientific implications. In the workshop, we recognized that it makes sense to apply a similar series of questions to other moments in the history of mathematics. In the end, we recognized that the interaction between scientific modes of thinking and the arts of thinking was both richer and more significant than scholars have previously understood. It became clear to us that this interaction deserved a much more thorough examination.

This volume presents some of the first results of our collective enterprise, and it concerns the sixteenth, seventeenth, and eighteenth centuries. Although neither the full range of our workshops nor the many conclusions of our research are contained here, the three essays do offer evidence of the importance of a re-examination of the role in science of rhetoric and the art of thinking.[2] As these three essays suggest, once we broaden our sense of rhetoric to include dialectic, logic, and the various other 'arts of thinking'—according to the norms of the early modern period—rhetoric becomes an important tool by which to dissect

[2] The contributions by Orna Harari, Alain Bernard and Catherine Goldstein will be published in 2007.

episodes in the history of early modern science. This tool allows us both to enter more thoroughly into the intellectual context of early modern agents and to place us more closely at the discourse level of the scientific work we wish to understand. Moreover, by studying the 'art of thinking' and recognizing its intersection with the history of mathematics, we are in a position both to reconsider the early modern criteria of scientificity and to grasp the process of the constitution of disciplines in their making. In the end, we begin to recognize the centrality of rhetoric—conceived indeed as 'art of thinking'—in the history of science.

Reconsidering Early Modern Science: Methodological Matters

The questions of our workshop were inspired by the historiographical 'air du temps'. In the past thirty years, historians of science—and especially historians of mathematics—have paid special attention to the context of production. The goal was to understand the formulations, priorities, and cultural concerns of the historical authors and texts. How were scientists educated and how did they manufacture their products? It will be helpful to clarify how our approach fits into the historiographical landscape.

In the beginning, neither 'Renaissance science' nor the 'art of thinking' were taken to be relevant to the historian of science. Consider the introductory lines to Alexandre Koyré's important essay on Renaissance science of 1949:

> …we all know, especially today, that the inspiration of the Renaissance has not been a scientific inspiration. The ideal of a civilization in the time that we call precisely the Renaissance '*of the letters and the arts*' is not by any means a scientific, but a rhetorical ideal. It is therefore extremely characteristic that the great logical reform that it has attempted—I think here of Ramus' logic—has been an attempt to replace logic's classical technique of demonstration by a technique of persuasion.[3]

Koyré's essay bears witness to the historiography employed in the mid-twentieth century. Although at the time his concern was to criticise Positivism, his essay reveals the *querelle* of the 'two cultures': for the historian of science, there was the culture

[3] A. Koyré, "L'apport scientifique de la Renaissance," in *Etudes d'histoire de la pensée scientifique* (Paris, 1966), 50 (my translation).

of the 'letters and arts', and there was 'science'. The two were considered radically different. For the historian of science, the point was to study the process of transformation that led to the accumulation of scientific ideas.

The methodologies of pragmatists like Thomas Kuhn and Giulio Preti were more culturally sensitive than those of Koyré and his contemporaries. Such historians of science rejected the assumption that scientific ideas 'accumulate' and endorsed the idea that the object of study included scientific theories (and their 'tensions') as well as scientific activities, their social relations and values. However, the distinction between the process of science and of scientific ideas was still active, and the Renaissance continued to be viewed in terms of an opposition of 'two cultures'. We find this attitude in the following statements by Giulio Preti:

> In a mostly literary civilisation, as that of the Renaissance, science will oscillate between ultra-utilitarian attitudes (techniques or even magic), on one hand, and attitudes of formal, theoretical purity (such as neo-Euclidism of geometry in the Renaissance), on the other. Both attitudes, however, renounced to a knowledge and conception of the world.[4]

In recent decades, historians of science have gone further in the contextualization of the productions of 'scientists' and attempted to reconstruct the places, circumstances, and conditions of the genesis of scientific theories. Some historians have endorsed the 'linguistic turn' of social history and applied this scholarly approach to the history of science.[5] This 'turn' has stressed the importance of 'discursive practices' as a means of constituting stable beliefs within a group of scientists and of privileging some beliefs over others.[6] Still other scholars have amplified this approach by analyzing the political economy of scientific theories;[7] or have studied historically the criteria of contemporary science—like the criterion of *objectivity* and the *attention* of the scientist, or the features of the *scientific persona*—to historical productions.[8]

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[4] G. Preti, *Retorica e logica* (Turin, 1968), 146.

[5] See S. Cerutti, "Le 'linguistic turn' en Angleterre: notes sur un débat et ses censures," *Enquête*, 5 (1997), 125-140.

[6] See, for instance, *The Literary Structure of Scientific Argument. Historical Studies*, ed. Peter Dear (Philadelphia, 1991).

[7] See the paradigmatic study by S. Shapin and S. Schaffer, *Leviathan and the Air-pump. Hobbes, Boyle, and the Experimental Life* (Princeton, 1985).

[8] See, in particular, L. Daston, "The Moral Economy of Science," *Osiris* 10 (1995), 1-6.

In the history of mathematics, Karine Chemla's project to look at the history of mathematics as a history of texts (as opposed to merely a history of mathematical ideas) nicely exemplifies this new approach.[9] The significant work of Catherine Goldstein goes even further. After calling attention to the importance of the 'mathematicians' skills',[10] Goldstein has focussed on theorems in order to explore in detail the process of constitution, reading, understanding, reconstruction and alternative demonstration of a single mathematical element. She argues convincingly for a vision of mathematics where, between the theorem and the historian and between the text and the interpretation, there is a continuum, which reproduces the very context of production and reading of the mathematical result.

Nick Jardine has returned to the question of the linguistic turn in the historiography of science when discussing the possible uses of the debate on *emics* and *etics*.[11] In the first, the actors' categories, the categories of the historical figures (equivalent to *emics,* in Jardine's terms), are the object of study. But in the more recent history of science, *etics*, which Jardine identifies with 'our categories', become indispensable to accurate analysis, to the extent that the latter's object is rather tacit knowledge and skills. Jardine's main point is the distinction between the traditional history of science, which depends on the history of ideas, and the new history of science, which tries to reconstruct tacit knowledge and skills.

The distinction between the actors' categories and our own categories as applied to the actors is relevant to the work presented in this fascicle, because rhetoric constitutes both the theory and the practice of an art. In this sense, while it is most interesting to understand the tacit rhetorical skill of a scientific author, we would best be helped in taking into consideration not only our own understanding of the rhetoric employed in scientific texts, but also the interplay between the rhetoric we see in action and the theory of rhetoric as explained in the treatises. Insofar as

[9] See in particular "Histoire des sciences et matérialité des textes. Proposition d'enquête," *Enquête*, 1 (1995), 167-180.

[10] One of the possible renderings of the expression *le métier* ('craftsmanship') is described in C. Goldstein "Le métier des nombres aux XVIIe et XIXe siècles," in *Eléments d'Histoire des Sciences*, ed. Michel Serres, (Paris 1989), 275-295.

[11] "Etics and Emics (Not to Mention Anemics and Emetics) in the History of the Sciences," *History of Science*, 42 (2004), 261-278.

rhetoric is both a tacit skill, but also an explicit doctrine described in appropriate treatises, it blurs our current methodological distinctions. This gives the historian a chance to reach historical 'facts'.[12] The tacit skill of rhetoric is, at least in part, reachable through the study of rhetorical doctrine and practice. The historian will be able to go beyond a traditional history of ideas insofar as the form or rhetorical garb of such ideas will have to be decoded. The attention to rhetoric is not a reduction of mathematics or science—including the science of history—to discourse, but a strategy of overcoming the *etic* language and of approaching the *emic*. Even if *emic* is by definition unreachable, the act of historicising rhetorical practice can provide hints concerning the *facts*, the constructs that are the objects of history.

A similar argument may be made for tacit mathematical skills[13]—which confirms our suspicion that to rethink the history of a science in terms of rhetoric forces us to examine the question of 'the two cultures' in its historical dimension and to allow for the possibility of an interplay between disciplines that are nowadays independent, or even opposed to each other.

Shortly before Kuhn and Preti offered their 'pragmatic' analyses, Perelman launched a program of studying arguments of scientific and extra-scientific sorts, which was based on the idea that any science has a rhetorical as well as a logical dimension. Only later did it become commonplace to talk about 'scientific discourse'. In response to this program, however, historians of science have been focussing only on the role of rhetoric in legitimating scientific disciplines; rhetoric as analysed in recent essays is taken as a way to articulate new discoveries in view of their reception and not, as we suggest here, as an argumentative structure possessing a cognitive role for the authors themselves. In fact, this part of 'science studies' presupposes a distinction between rhetoric and science that is typical of the period succes-

[12] I am referring to the notion of 'facts' as units of historical knowledge. Its history and its relevance to contemporary historiography are developed in S. Cerutti and G. Pomata (eds.) *Fatti: storie dell'evidenza empirica*, special issue of *Quaderni Storici* 108 (2001), no. 3.

[13] See by E. Brian : «Le livre des sciences est-il écrit dans la langue des historiens?,» in Bernard Lepetit (ed.), *Les formes de l'expérience. Une autre histoire sociale* (Paris, 1995), 85-98. See also M. Halbwachs, A. Sauvy, et al., *Le Point de vue du nombre (1936)*, critical ed. E. Brian et M. Jaisson, with contributions by W. Gierl, J.-C. Marcel, J.-M. Rohrbasser et J. Véron (Paris, 2005).

sive to the early modern age. Our task is therefore to historicize this distinction, in an attempt to discern precisely which type of rhetoric was involved in science with respect to the various 'rhetorics' available at the time, to investigate analytically the history of rhetoric in its relation to science, and to study the extent to which rhetoric could actually work as a strategy of conception in science itself. The historiography of science—and sixteenth-century studies in particular—permit us to return to a time for which today's opposition between the two cultures was not yet valid. In this respect, it might even be said that the reform of the sciences had been first conceived of as a reform of their rhetoric.

Our endeavours depend on some aspects of micro-history because of the need to understand both the dynamics of a precise context of knowledge and of the actors involved. Importance is therefore given to the actors' point of view and to their vision of their own experience. Carlo Ginzburg has recently drawn attention to the important cognitive role of ancient rhetoric as the realm of enthymemes, stressing its inclusion of extra-logical knowledge (implicit media) into the argument.[14] His purpose was to illuminate several unknown dimensions of proof and logical rigour in the science of history through the centuries.

Our essays are furthermore intended to make a contribution to the history of the 'scientific revolution', to the history of mathematics with particular attention to mathematical practices and traditions, but also to the history of science insofar as it sheds light on little-studied aspects of mathematics and on the way in which algebra became a candidate for the replacement of rhetoric in some of the mixed sciences.

The methodology that informs the essays collected here has grown out of the methodological lessons taught, on the one hand, by previous historians of science and, on the other, by our own workshop experience of studying the boundaries between mathematics and rhetoric. For us, algebra has offered a particularly poignant example: once we focussed on its place in the sixteenth century, we found that 'the two cultures' disappeared. On the contrary, it appeared that the reform of mathematics and science in the sixteenth century was first conceived of as a reform of the 'art of thinking'.

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14 C. Ginzburg, *Rapporti di forza. Storia, retorica, prova* (Milan, 1999).

Sixteenth-Century Algebra and the Importance of Rhetoric

The history of algebra in the sixteenth century offers striking evidence of our main point, namely that an investigation of the relations among science, rhetoric, and the art of thinking can offer important help in deciphering the history of science.

Algebra is in this respect particularly interesting, because from the perspective of the sixteenth century, it was both old and new—an ancient art in the process of being reinvented as a discipline. During its transformation into symbolic algebra (between 1543 and 1591), it absorbed in its categories and in its logical functioning many aspects of the new dialectic, or logical rhetoric, which flourished at the time. The common goal of the new dialecticians was to rearrange into a single 'art of thinking' all the arts of discourse and all sorts of reasoning ranging from persuasion to proof. These thinkers were concerned as much with the search for arguments (invention) as with the deduction of conclusions (disposition, judgement). Once algebra had achieved the status of a discipline, it assumed a series of special tasks that were proper to dialectic, insofar as algebra provided the means to formulate theoretical problems in a clear and concise way and even to solve them, first by an explicit formulation in letters (A, E, etc.) and later by a systematically applicable procedure.

In the sixteenth century, the new dialectic had developed as an art that rendered possible a new encyclopaedia of the sciences ranging consistently from mathematical sciences to jurisprudence.[15] The underlying goal of this encyclopaedic project was the creation of an art that would allow all forms of knowledge to be arranged in such an orderly fashion that they could be learned more effectively and their fundamental interconnections could be recognized. Algebra, after its transformation into symbolic algebra, took the form of an auxiliary art with applications in geometry, optics and mechanics. It thus became the 'art of thinking' in the mathematical disciplines and was taken to carry to completion the previous project of the unification of the sciences. For symbolic algebra could be used in all disciplines as an organizing principle and guarantee of their coherence and rigor.

[15] The term 'encyclopaedia' is here meant to refer to that renewal of all knowledge based of a new foundation in dialectic, as explained in the first essay of this fascicle.

All of this means that one must look in a new way at the content of sixteenth-century mathematical texts and textbooks. At the time, mathematical texts came in two main genres—theoretical (versions of Euclid, Boethius, Sacrobosco's/Holywood's *Algorismus* and the *Sphaera*) and practical (abacus arithmetic and practical geometry, as taught at abacus schools). With the advent of printing, two other genres were introduced: *Institutiones mathematicae*, which became popular in the new humanist colleges, and arithmetic and algebra treatises combining the algorism with some techniques of commerce and or algebra, quite in keeping with the general move of vernacular texts describing an art. In fact the relation between algebra and rhetoric—as an 'art of thinking'—in the early modern period is best understood in the context of Aristotle's *Posterior Analytics* (71a1): "All teaching and all intellectual learning come about from already existing knowledge." Early modern thinkers were keen to organize knowledge not just because they considered the organization of knowledge a good thing in itself, but also because they believed that to organize knowledge was to create it. Seen from this perspective, to write a mathematical text in encyclopaedic form, as happens in the type of *Institutiones mathematicae*, meant to stress the formal, ordering function of mathematics. Even where recent mathematical results were mentioned, the overall purpose was still to anchor the new in the old, to assume the new as a part of the old, or to explain the old in terms of the new. The typical style of sixteenth-century mathematical texts can therefore be seen as the fulfilment of an intellectual program, rather than as the product of intellectual inertia. That is, the mode or style of presentation is itself a part of the epistemological program. Indeed, as I have tried to show, some of the most technical mathematical innovations of the sixteenth century (as, for example, the systematic use of letters in equations) emerged in the context of rhetorical consideration and presentation.[16]

16 G. Cifoletti, "La question de l'algèbre. Mathématiques et rhétorique des hommes de droit dans la France du XVIe siècle," *Annales. Histoire, Sciences sociales*, 6 (1995), 1385-1416.

Rhetoric—a Re-evaluation

But what kind of rhetoric played this role? What did sixteenth-century thinkers mean, in fact, by 'rhetoric'? The texts suggest that rhetoric is roughly conceived as a strategy of thinking, a strategy employed before giving shape to a structured and polished text or public speech. The object to be organized is thought itself. It is the process of giving shape to discourse, where the latter is articulated thinking. For many thinkers in the sixteenth century, this articulation is strictly dependent on one's relation to one's mother tongue. Lorenzo Valla had begun to reflect on the link between thinking and the language in which it occurred, and he had done so with reference to Cicero. Cicero had taken it upon himself to transfer Greek philosophy into the Latin language and culture. He had first asked, and then solved, the question regarding the connection holding between Greek language and philosophy. To imitate Cicero meant to give modern languages a new impulse by using them in philosophy, the sciences and the letter, in avoidance of Latin as the language of creative writing and speaking. By the same token, the logic of the new encyclopaedia was to replace Aristotelian logic, appropriate only for his own encyclopaedia. This new logic had to originate from the grammar of the mother tongue and its inherent rhetoric—this logic they called by the name of 'dialectic'. According to Lorenzo Valla, Rudolf Agricola and Peter Ramus as well as for that larger movement that developed in their wake, most of the ancient art of rhetoric was incorporated in dialectic and was supposed to make explicit the rules of thought and demonstration. This was done mainly in two ways: first, by interpreting invention and disposition, traditionally parts of the *oratio*, as the key constituents of any argument—and hence of any logic; and, secondly, by considering dialectical—or abbreviated—syllogisms, called enthymemes, a valid type of argument.

By unifying the arguments of all sciences Valla, Agricola and Ramus were consciously building a new encyclopaedia on the basis of the new vernacular grammars. The challenge presented by these 'natural' languages and their respective grammars suggested to these authors the afore-mentioned new agenda of constructing a logic that was independent of 'Aristotelian' logic and an encyclopaedia that was independent of 'Aristotelian' ontology. This development can be translated into our own terms: while the scansions of the Greek and Latin grammars led to a predicative logic with monadic relations, the new grammars pointed to more general relations and to propositional logic.

Rather more curious to modern eyes may appear one of the supposed results of this logical reform, namely the return to the 'authentic' Aristotle. For, many authors, including Ramus, formulated their most audacious theories while participating in the debates on the interpretation of Aristotle's logical works. Whereas the medieval logical tradition had provided the target for all Renaissance commentators, because humanistic philology had shown the complex stratification of theories and practices involved in that tradition, two main lines prevailed among Renaissance commentators: the first stressed the need to recover an Aristotelian orthodoxy in logic, and used Galen's logic to define it; the second underlined the compatibility between Plato and Aristotle, and used Proclus' theory to articulate it. Both currents not only claimed to restore the 'authentic' Aristotle, but also to imitate him, in the same way as they claimed to be imitating Cicero by reproducing his method of constructing a science of the basis of their own language: what was Latin for him, as opposed to Greek, was for the moderns their mother tongues, as opposed to Latin.

By approaching Aristotle in this conciliatory manner, sixteenth-century philosophers could claim to be combining the new with an authoritative tradition in the *concordantia* between Plato and Aristotle. Such a concord had been pursued by neo-Platonic philosophy a thousand year earlier, notably by Proclus. Proclus had introduced the notion of *mathesis universalis* to describe a science of the pure human mind, generated exclusively by the elementary art of thinking, and as the site where Plato's ideas and Aristotle's universals coincided. Neo-Platonic texts were rediscovered and published in the sixteenth century, as was Proclus' commentary on Euclid's *Elements*. Ramus' project in dialectic converged with the view of mathematics promoted by Proclus' texts and with a valorisation of the medieval art of algebra. The important role assigned to Platonism in this conciliatory project suited the tastes and interests of Platonic court academies as well as many members of the university community.

The 'art of thinking' was seen as a necessary part of this concord, insofar as it involved a basic knowledge of humankind, and an understanding of the light of reason as a property of each individual. With respect to Aristotelian logic, the new dialectic and the related 'art of thinking' had the advantage of spelling out the rules of reasoning as found in the natural languages, but also the logic one presumed to have characterized the universal language spoken before the Tower of Babel incident.

It appears therefore that the transformation of algebra into symbolic algebra occurred in connection with the application of the new vernacular dialectic to the reformulation of the arts into scientific disciplines, with the structuring role of rhetoric on the encyclopaedia being transferred to symbolic algebra. This process shows a complex process of negotiation between various traditions. It also suggests the possibility that a similar cross-fertilization between the 'art of thinnking' and mathematics may have occurred in other periods and contexts.

Beyond Algebra and the Sixteenth Century

Because algebra concerns the giving of shape to equations and because it stresses the role of notation, procedure, and syntax, it is maybe not surprising that there exists a rich relation between algebra and rhetoric. But what about other areas of mathematics and the sciences? As our workshops made clear, other historical contexts and other mathematical fields offer equally rich examples of the dynamic relations between rhetoric, the 'art of thinking' and science. Concerning Antiquity, it is necessary to stress the role of the strictly rhetorical tradition apart from the better-studied tradition of demonstration. A few authors have especially dealt with this topic, in particular G.E.R. Lloyd and R. Netz. According to Lloyd, mathematical *apodeixis* is partly a development of the rhetorical *epideixis*. [17] This view agrees with the characterization of Greek culture as agonistic and opens the way to a comparative study of different forms of demonstration in ancient Greece. Reviel Netz has chosen to start from the same picture, but with a clearly hierarchical structure of arguments: "There is, first, an activity of great prestige for the Greeks: that of making compelling arguments. And there is one type of argument which is more

[17] See G.E.R. Lloyd, *Demystifying Mentalities* (Cambridge, 1990), and "Theories and Practices of Demonstration in Galen," in M. Frede and G. Striker (eds.), *Rationality in Greek Thought* (Oxford, 1999), 255-278.

compelling than others, which leaves less room for controversy than others. This is mathematics."[18] Alain Bernard, in turn, has begun to explore Greek mathematical arguments not for their compelling nature but as part of contests among rhetoricians. Bernard's approach requires that we introduce a classification within the set of mathematical arguments. Some of them developed in a context of other kinds of arguments and are therefore perhaps slightly less compelling than those appearing in famous demonstrations; but they are typical of the rhetorical style and quite useful in problems and geometrical analysis.

When placing Pappus in the larger context of mathematics in late Antiquity, Bernard has shown that the contemporary rhetoric –both as common practice and as theory—entered into the constitution of mathematics, noting in particular the special role played by analysis and invention.[19] Orna Harari has furthermore reconstructed the presuppositions which, for Aristotle, rendered classical syllogisms incompatible with mathematical demonstration.[20]

Catherine Goldstein has taught us how to follow mathematical objects and texts from their constitution through to their apparent permanence in time. This permanence allows mathematicians to look at a theorem in a variety of versions, for the later historian to study.[21] In particular, Goldstein has applied detailed textual analysis to a corpus of problems in order to highlight the relations between the form of statements and their various versions in different mathematical and disciplinary contexts.[22] She has also worked on the articulation of method, writing (*mise en écriture*) and heuristic in early modern science.[23] At our workshop, Ca-

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[18] Reviel Netz, *The Shaping of Deduction in Greek Mathematics. A Study in Cognitive History* (Cambridge,1999), 310.

[19] Alain Bernard, "Sophistic Aspects of Pappus's Collection," *Archives for the History of Exact Sciences*, 57 (2003), 93-150; "Comment définir la nature des textes mathématiques de l'antiquité grecque tardive? Proposition de réforme de la notion de 'textes deutéronomiques'," *Revue d'histoire des mathématiques*, 9 (2003), 131-173.

[20] O. Harari, "The Concept of Existence and the Role of Constructions in Euclid's Elements," *Archive for the History of Exact Sciences*, 57 (2003), 1-23. See also *Knowledge and Demonstration: Aristotle's Posterior Analytics* (Dordrecht, 2004).

[21] C. Goldstein, *Un Théorème de Fermat et ses lecteurs* (Saint Denis, 1995).

[22] Eadem, "L'arithmétique de Fermat en contexte, L'arithmétique de Pierre Fermat dans le contexte de la correspondance de Mersenne : une approche microsociale," *Sciences et techniques en perspective*, IIe série, 8 (2004), 14-47.

[23] Eadem, "L'expérience des nombres de Bernard Frenicle de Bessy," *Revue de Synthèse*, *4e série*, n° 2-3-4 (April-December, 2001), 425-454.

therine Goldstein presented her research on Frenicle de Bessy, and in a forthcoming paper she examines in details how the views expressed in Francis Bacon's *Novum Organum* on method, the constitution of evidence and proof, and the composition of a heuristic influenced theoretical mathematicians like Frenicle de Bessy in the context of a mathematical activity which was focussed on problems and the effective construction of their solutions.[24] In their respective roles as the arts of discourse and discovery, rhetoric and dialectic were here tightly linked, providing the possibility of writing a method of mathematical invention (or of exclusions, as in Frenicle), distinct from the classical treatises of the Euclidean type. The main role for the discovery of principles falls to a systematic induction, which is based on reasoned observation and requires specific training for one's attention, senses and mental processes. It first ascends to the axioms, then returns to the particular results necessary for the operationality of human activities. In this double process, the construction and use of a series of tables hold a prominent place. Testing is proof, with truth appearing explicitly as an interactive practice.

In *Leibniz's Metaphysics*, Christia Mercer has shown that Leibniz developed his metaphysics by reconciling the views of several philosophical traditions.[25] He found the fundamentals of his 'metaphysics of substance' in Aristotle and the elements of his 'metaphysics of divinity' in the Platonism of Plotinus and other Platonists, while major parts of his mechanical physics were taken from Descartes and other 'moderns'. In her book, Mercer argues that Leibniz' means of obtaining philosophical truth involved a strategy of combining ideas and intellectual elements from a wide range of sources. As a member of our workshop, Mercer came to view this conciliatory program as a part of a more general rhetorical strategy of leading readers towards the truth by engaging them in a rhetorical process of discovery. Although this conception of rhetoric places the burden of truth-finding on the reader, Leibniz is willing to help the truth-seeker by offering rhetorically subtle intellectual aids. It thus seems as if rhetoric played a role in both the evolution and the presentation

[24] Eadem, "Writing Mathematical Experience: Method according to Frenicle de Bessy" (forthcoming).
[25] Christia Mercer, *Leibniz's Metaphysics, its Origins and Development* (New York, 2001).

of Leibniz's thought.

Eric Brian, in *La mesure de l'Etat*, has studied the transmission of mathematical craftsmanship through teaching very closely.[26] For some mathematical areas of social relevance such as probability, statistics or demography, teaching possessed certain peculiar features involving tables and formulas. Brian has similarly shown the importance, in the tradition that connected mathematics and rhetoric, of Condorcet's project of a universal language. Condorcet wrote between 1793 and 1794 several texts connected to the vast project of his *Tableau historique du progrès de l'Esprit humain*, which is known only in its shorter version, *l'Esquisse*, written since 1795. Extensive research, mostly at the library of the Académie des Sciences, has allowed a team of researcher including Eric Brian to find important parts of the unfinished *Tableau*.[27] In the present article, Brian examines not so much Condorcet's project of a universal language in itself, but rather an important example of its realisation, the 'analysis situs' or *géométrie de situation*, one of the many disciplines belonging to the encyclopaedia with which the universal language successfully deals with, and which includes algebra, geometry, mechanics, chemistry, and the moral and political sciences. *Analysis situs* is a mathematical art of thinking: Leibniz invented it as a part of his reflections on logic and calculus, while Condorcet reinterpreted it in the tradition of French symbolic algebra.

By Way of Conclusion: History of Mathematics and its Philosophy

Henk Bos has recently raised the question of the historicity of mathematics.[28] He did so, as it were, in spite of himself. At the outset, his purpose was to respond to some historical cases. There are examples, he claims, for which the emphasis on truth is not a good guide for the historian of mathematics. The reason is

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[26] Eric Brian, *La mesure de l'Etat. Administrateurs et Géomètres au XVIIIè siècle* (Paris, 1994).

[27] Condorcet, *Tableau historique des progrès de l'esprit humain. Projets, Esquisse, Fragments et Notes (1772-1994)*, ed. Jean-Pierre Schandeler, Pierre Crépel, et al. (Paris, 2004).

[28] Henk Bos, "Philosophical Challenges from History of Mathematics," in T. Hoff Kjeldsen, *New Trends in the History and Philosophy of Mathematics* (Odense, 2004), 51-66.

that truth itself is context-dependent: in general, the translation of a context of truth into a modern one does not preserve the truth of each statement. Bos writes:

> In both examples historians interpret and assess an achievement by attempting to reconstruct concepts from past mathematics. To do so they find or create a consistent modern mathematical concept or structure which best fits the recorded statements from the past.[29]

This has been the method commonly used by historians of mathematics, and, as Bos points out, its assumption is "that there actually was a concept, and that this concept may be represented by a unique consistent modern analogue." Sometimes this approach works, sometimes it does not. When it works, the past concept has an 'elegant' modern analogue. When it does not, there is no common ground in modern mathematics on which the question of the past concept, for instance the validity of Cauchy's theory with respect to his notion of continuum, can be decided. The question is the uniqueness of the modern equivalent, that is, the question of whether by the translation we get only one entity. For, according to Bos, "Platonic conceptions of mathematics do not allow different understanding of the same mathematical entity, and hence cannot deal with the development of mathematical thought."[30]

However, it should be noticed that to allow different understandings of the *same* mathematical entities is a strong assumption, which could also be called Platonic, insofar as it implies the existence of a permanent realm of mathematical entities. In fact I believe that historians of mathematics have used this as a working hypothesis. In any case, to assume the translatability of earlier mathematics implies at least some tolerance towards a variety of understandings of the same mathematical entities and of different formulations through time, but under the condition of a certain uniqueness of features which would lead univocally to a modern equivalent. Bos' conjecture is that to see the mathematical world as constituted by tasks instead of Platonic objects might be of some help in cases where there is no modern analogue, or where there are several candidates and where it is not possible to decide among them.

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29 Ibid., 51.
30 Ibid., 65.

In a book on Fermat's method for maxima, minima and tangents, I have examined the question of what can be considered a true statement within a certain, time-dependent, mathematical context.[31] Fermat asserted that his method applied not only to all maxima and minima, but also to tangents, centers of gravity, asymptotes etc. For some cases, Fermat explained the procedure, for others he at least suggested the direction to be taken, while there are yet other cases in which the procedure is entirely open to conjecture. While we know that Fermat was inclined to state conjectures as theorems, we also know that his conjectures tended to be sound. My book makes an attempt at verifying his method for maxima and minima. What appears quite clearly is that Fermat's statements are relative to the group of problems he had at hand. For instance, his method was sound for all the curves he worked with, even though he had himself contributed to extend the domain of curves.

But returning once more to Henk Bos, his recent philosophical statements emerge, it seems, from his great familiarity with the history of early modern mathematics. Interestingly, his view of mathematics has striking similarities to that of sixteenth-century mathematical authors as described above. According to Bos, there are a few 'basic tenets' of a philosophy of mathematics, if mathematics is indeed understood as the performance of self-imposed tasks. These are:

– All mathematics was, is and always will be incompletely understood.
– All mathematical concepts are fluid.
– For many historical periods in which mathematical activity can be traced, the image of mathematics as centred on objects and propositions whose existence and truth are safeguarded by logical proofs is not applicable.
– The historical perception of mathematics occurs through both the actions of mathematicians and their mathematical results; as the latter are often difficult to recapture, it is essential to develop an understanding of the former.
– The actions of mathematicians are to be understood as performing self-imposed tasks according to self-created criteria

[31] G.C. Cifoletti, *La méthode de Fermat, son statut et sa diffusion. Algèbre et comparaison de figures dans la méthode de Fermat* (Paris, 1991).

for quality control.

- The use of terms as 'truth', 'proof', 'rigour', 'exactness', 'purity', 'legitimacy', etc. by mathematicians generally indicates a conviction that the mathematical endeavour can produce knowledge of a particularly absolute quality. However, the historical analysis of mathematical developments should not be based on this conviction.

Sixteenth-century mathematicians realised that there existed an immense wealth of mathematical knowledge in the classical and less classical past, to be rediscovered and organised. To understand mathematics was as much to retrace, say, Apollonius' argument in a rediscovered text as to add a new solution to an arithmetical textbook. The philological attitude towards the classical heritage is the way in which early modern mathematicians held in common, and in fact invented the first three tenets and elaborated their mathematical activity on that basis.[32]

The view of mathematics as the performing of self-imposed tasks seems to have been widespread through the centuries, but acquired a more dominant role in certain periods, for instance in late Antiquity and in some Arabic traditions. Clearly, the sixteenth century was also one of those moments; for any survey of mathematical books of that century will document that mathematical disciplines possessed the character of problem-solving activities.

As to algebra in particular, Bos' 'reduction' of mathematics to tasks confirms the very notion of algebra inasmuch as it was identified with the project of solving all problems. Late Antiquity—Pappus, Diophantus, etc.—provided geometrical and numerical problems, while the abacus schools provided commercial arithmetic problems and several solution techniques. Geometrical analysis, Diophantus' analysis, and the medieval algebraic techniques appeared to sixteenth-century mathematicians intrinsically connected, in spite of their diversity: all the problems pertaining to these three traditions did not require demonstrations and constructions, but solutions, for example so as to determine a quantity, number or line, under given conditions. Mathematicians

[32] On the philological attitude towards the classical heritage, see P.L. Rose, *The Italian Renaissance of Mathematics. Studies on Humanists and Mathematicians from Petrarch to Galileo* (Geneva, 1975), and J. Morse, *The Reception of Diophantus' 'Arithmetic' in the Renaissance* (PhD thesis, Princeton University, 1981).

started to visualize problems of those three fields in the form of equations.

There is another, intrinsic reason why sixteenth-century symbolic algebra fits the 'basic tenets': the idea of general quantity itself presupposes that the unknown (the root) varies first on different sorts of quantities, and secondly on a set of quantities of the same sort, while only one of these quantities is the solution. In this sense, algebra itself deals with a non-categorical concept of general quantity, or with the variation of concepts: it was invented well before the notion of function, in order to master the variation in contents of a concept. May we say that the variation that has been mastered is the change of intension with respect to a unique or categorical extension? If this is the case, we should also include a chronological dimension, i.e. historicity. What is at stake is how sixteenth-century mathematicians perceived the mathematical entities that they learned to know through the works of their Greek and Hellenistic predecessors. My working hypothesis has been that symbolic algebra made sense in the context of late humanism, i.e., in connection with philological activities and consistently with them. Jacob Klein in 1934 argued that Viète and his symbolic algebra radically changed the understanding of number with respect to the Greek view: the new notion introduced 'intentionality' in number, from the number of things Viète came to number.[33] I believe that this view can be translated into a research program for the historian of mathematics. My suggestion is to see the intentionality as the way in which sixteenth-century mathematicians conceptualized their relation to the mathematical classics. The classical heritage constituted a domain of investigation, including a great amount of mathematical tools and a universe of problems, some of which had been solved, while others were still in need of a solution. First, this realm of mathematics constituted the field of action for the new algebra: in order to test universality for any statement, it was necessary to make sure that it held true in that universe. Secondly, the very existence of this tradition assured the ontological basis for the meaning of the new algebra. The origin of this new algebra was essentially philological. This fact deserves to

[33] J. Klein, *Greek Mathematical Thought and the Origin of Algebra* [1934-36], transl. Eva Brann (Cambridge, Mass., 1968).

be taken seriously: we must accept that early modern algebraists saw their mathematical research activity as connected to their philological activity of reconstruction.

Finally, we ought to acknowledge that in order to validate this reconstruction, they searched for its origin in both ancient texts and their own minds. The key for understanding this attitude to the (mathematical) past lies in the theory of imitation.[34] Historicity was accepted as being part of mathematics itself. Algebra in particular had been shaped only in the secular Arabic tradition, and the challenge for European algebraists was to see it as part of the classical tradition of mathematics by interpreting it as the calculus of general quantity.

Mathematics was seen as a discipline developing from the faculty of counting. While there was a sense that the whole of mathematics rested on the processes of the human mind, *mathesis* was considered as fundamental to human knowledge and as proper to human apprehension of the world as grammar was to any modern language. Mathematics, therefore, functions independently of proofs, at least of the syllogisms of the Aristotelian scientific sort—as we shall document in the first essay of this collection. The mind was enough of a guarantee for the existence of mathematical facts. In the same line of thought, problems were considered more important than theorems, and algebra as the science of solving problems or tasks. These can be great ancient geometrical problems, all the problems concerning numbers, trigonometric and astronomical problems, and of course all abacus problems. Given that algebra allows mathematicians to 'solve all problems', all the emphasis is on finding the right form. The way to solve problems becomes a highly 'rhetorical' practice, by putting the problem, any problem, into an equation.

[34] JoAnn Morse has applied this theory to the reception of Diophantus. I extend the application of this idea in early modern algebra in my "La question de l'algèbre" and in "The Algebraic Art of Discourse. Algebraic *Dispositio,* Invention and Imitation in Sixteenth-Century France," *History of Science, History of Text,* ed. K. Chemla (Boston Studies in the Philosophy of Scienc 238) (Dordrecht, 2004), 123-135.

FROM VALLA TO VIÈTE: THE RHETORICAL REFORM OF LOGIC AND ITS USE IN EARLY MODERN ALGEBRA

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Abstract

Lorenzo Valla's rhetorical reform of logic resulted in important changes in sixteenth-century mathematical sciences, and not only in mathematical education and in the use of mathematics in other sciences, but also in mathematical theory itself. Logic came to be identified with dialectic, syllogisms with enthymemes and necessary truth with the limit case of probable truth. Two main ancient authorities mediated between logical and mathematical concerns: Cicero and Proclus. Cicero's 'common notions' were identified with Euclid's axioms, so that mathematics could be viewed as core knowledge shared by all humankind. Proclus' interpretation of Euclid's axioms gave rise to the idea of a universal human natural light of reasoning and of a *mathesis universalis* as a basic mathematics common to both arithmetic and geometry and as an art of thinking interpretable as algebra.

In this paper, I employ 'rhetoric' in two senses each of which has its own historical roots. According to the first, 'rhetoric' is the new dialectic emerging from the rhetorical reform of logic, and according to the second, it is the art of thinking.

In Lorenzo Valla's reform of logic, rhetoric became a new dialectic, which was conceived as a logical discipline springing naturally from the vernacular languages. In the sixteenth century, Ramus took this reform further by regarding the first two parts of the art of oratory[1], invention and disposition**,** as a true art of thinking. Ramus called these two parts *eloquence*, while he reserved the term 'rhetoric' for simple 'elocution' and 'pronunciation'. He promoted the linkage of eloquence with philosophy, so that eloquence could become the intellectual tool for sixteenth-century French algebraists to interpret their 'art' in a larger context and to transform it into a general and effective discipline. Later, eloquence assumed the name of 'art of thinking', becoming famous in the works of Arnauld and Nicole, Ramus' late heirs.

[1] For Quintilian oratory included invention, disposition, elocution, memory, pronunciation (*Institutiones Oratoriae* I, 22).

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The distinction, right at the outset, between these two aspects should help to handle a terminology that is inherently unstable—each of the authors considered proposes at least one distinctive interpretation of these terms—and to manage this category, which carries with it both continuity and discontinuity. The striving for an art of thinking, over and above the striving for a method is characteristic of the Renaissance, although in different ways it was pursued also in later centuries. This distinction furthermore allows us to perceive the role of rhetoric not as peripheral or extrinsic, but as constitutive of science.

Recently, one of the finest interpreters of Renaissance science, Nancy Siraisi, has written:

> The place of oratory and epideictic rhetoric in Renaissance academic medicine cannot be simply dismissed as peripheral to its "real" enterprise, whether that enterprise is conceived as primarily professional, scientific, philosophical, or healing.[2]

Siraisi has investigated the place of rhetoric by means of a close reading of orations, encomia, prolusions and other 'rhetorical productions' concerning medicine. Her sources, however, have led to a radical limitation of the extension of the place she assigns to rhetoric, for she continues:

> This is not only because some surviving orations express a scientific conviction or intellectual commitment or recount a biography that makes them well worth studying. Many others are essentially conventional or insignificant in content. But the character of medical orations and encomia is one more sign among many that the self-image of the learned medical profession then incorporated attributes characteristic of a Renaissance humanist discipline as well as those of a technical or scientific profession.

'Intellectual commitment' or 'factual information': these two interpretations alone would be of immediate use for us. But Siraisi also suggests to look for a sense beyond the conventional content. What is at stake, unexpectedly for us given the way we divide the domain of intellectual labor, is the very definition of the profession. To go even further: sometimes perceived 'dead ends' contain traces of a rhetoric whose use nowadays escapes us—traces of an 'emic' rhetoric, which can become comprehensible to us only

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[2] Nancy Siraisi, "Oratory and Rhetoric in Renaissance Medicine," *Journal of the History of Ideas*, 65 (2004), 191-211, at 204. According to Aristotle's *Rhetoric* I, 3, *epideictic* is the genre of rhetorical discourse in which the public (as opposed to the judge—*judiciary*—or the town—*deliberative*) judges the talent of an orator.

if we look at its use in a new way.[3] I propose to take rhetoric, in accordance with our introduction, as an art of thinking, and the rhetorical acts that we study as traces of this art of thinking in action. The special topic here will be rhetorical demonstration, or enthymeme. This paper will provide some examples of this striving towards an art of thinking, for the search of a new logic in the recovery of classical geometry, and for the connected enterprise of re-founding algebra in the Renaissance.

The French Algebraic Tradition

In my previous work I have tried to show that a group of mathematicians, Jacques Peletier, Petrus Ramus, Guillaume Gosselin and François Viète, were representatives of a unique new kind of algebra.[4] This "French algebraic tradition" provided the background for Viète's symbolic algebra. On the basis of a comparison of French algebraic texts with Italian and German counterparts and of an examination of the publishing context (mathematical and otherwise), I suggested a two-phase periodization. Jacques Peletier stands for the introduction of the abacus tradition (elementary commercial arithmetic) and algebra at the Court. Peletier's algebraic program was connected to his theory of rhetoric (poetry, oratory, dialectic). He published a genre of texts devoted to algebra in the vernacular, thereby promoting French as a scientific language. Rhetorical criteria shape his *L'Algèbre* and accentuate its structure and theory (that is, definitions and demonstrations). Peletier introduced innovations into Cardano's and Stifel's treatments of equations with several unknowns. In the second phase, style and content changed. Guillaume Gosselin is representative of this period. We find textbooks and manuals shifting focus from problems and questions to equations and classification. The purpose of algebra becomes the solution of equations. Problems

[3] I use 'emic' in the sense of Nick Jardine, "Etics and Emics (Not to Mention Anemics and Emetics) in the History of the Sciences," *History of Science*, 42 (2004), 261-278, as discussed in the Introduction to this fascicle.

[4] See in particular G. Cifoletti, *Mathematics and Rhetoric. Jacques Peletier, Guillaume Gosselin and the Making of the French Algebraic Tradition* (PhD thesis, Princeton University, 1992); and "La question de l'algèbre. Mathématiques et rhétorique des hommes de droit dans la France du XVIe siècle," *Annales. Histoire, Sciences sociales*, 6 (1995), 1385-1416.

are no longer addressed as such, but conceived, in their most general form, as the corresponding equations. Algebraists connect with a milieu of jurists already prominent in politics and scholarship. This erudite milieu favors the recovery of Diophantus' *Arithmetic*, which provides a set of theoretical problems concerning numbers. Whereas Diophantus had solved problems by transforming them into numerical cases, Gosselin now reached a general solution by transforming them into equations. Gosselin's notation made all the difference. The algebraists created an illustrious genealogy for algebra, deriving it from Greece (Diophantus) rather than from the abacus schools. Three features of this tradition (late abacus algebra, rhetoric, and genealogy) can be traced back to Italian humanism: Cardano's and Tartaglia's algebra, *imitatio* (translation to a new vernacular learned culture), and the construction of a history for the discipline. The French algebraists radically transformed all three. Their *translatio* authorized them to abandon the last ties to the medieval tradition and to build a new discipline that they could depict as national. The adoption of this discipline by the legal élite was facilitated by the rhetorical interpretation of logic developed in Paris at the same time. This provided a theoretical frame within which generalized algebraic problems could be seen as Cicero's *quaestiones infinitae*, that is, as scientific questions.

From this, it seemed to me that one could conclude that, first, a French algebraic tradition existed before Viète and, second, that this tradition developed in connection with changes in the rhetorical tradition.

The Rhetorical Reform of Logic: Proclus and the Mathematical Practitioners

It has already been mentioned that the new dialectic sprang from the modern vernacular languages; this was the main aspect of Lorenzo Valla's foundation for a new encyclopaedia.[5] Its bases were those primary arts that are common to all human beings, such as the use of grammar. Having no declensions, the grammar of romance languages does not model itself on Aristotelian logic

[5] See S. Camporeale, *Lorenzo Valla. Umanesimo e teologia* (Florence, 1972), and *Umanesimo, riforma e controriforma* (Rome, 2002).

and can therefore not rest on its ontology. The traditional scheme of subject, copula, and predicate assigned a hierarchical relation to verb and noun. This hierarchy was now replaced by a scheme taken to fix only a functional relation.[6] As a consequence, it became necessary to disentangle logic from Aristotelian ontology. For this purpose, the reform of scientific demonstration played an important role. Enthymemes or rhetorical syllogisms, which Aristotle had considered to be intrinsically uncertain syllogisms, were now reinterpreted as syllogisms lacking one evident premise, following Quintilian's definition, but as being otherwise perfectly adequate to the new functional grammar and logic and as being actually more typical of common uses of demonstration.[7]

In fact, a new reading of Cicero and Quintilian was the theoretical basis for Valla's new encyclopaedic system, which was built on 'common notions' (*notiones communes*). This term had been introduced into Latin by Cicero as a translation of a Stoic concept. In his own theory of truth, Cicero shared Carneades' sceptical theses, agreeing to maintain some innate basic truths ('common notions') on which to build the most warranted and indispensable knowledge; this knowledge, however, could only aspire to certainty if it followed some formal criteria and developed according to logical rules. [8] Cicero added to Carneades' theory of truth as probability his own eclectic agenda, which had developed out of the teaching he had received in Greece from Antiochus of Ascalon. The latter did not admit absolute differences among philosophers, at least between Plato and Aristotle, except in the words they used. This is the famous *concordantia* which would become so important for sixteenth-century theology, philosophy and science.[9]

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[6] This crucial point has been made for Ramus' grammar by P. Magnard, "L'enjeu philosophique d'une grammaire," *Revue des sciences philosophiques et théologiques*, 70 (1986), pp.3-14. The background to Magnard seems to be Ernst Cassirer's *Substanzbegriff und Funktionsbegriff: Untersuchungen über die Grundfragen der Erkenntniskritik* (Berlin, 1910).

[7] G. Cifoletti, "Matematica e fatti scientifici: polarità e simmetrie. Il ruolo degli algebristi francesi del Cinquecento nella costruzione dei fatti scientifici," *Quaderni storici*, 108 (2003), 771-797.

[8] Carneades, who flourished in Athens in the second century B.C., was the main representative of the sceptic Academy.

[9] See C.B. Schmitt, "'Prisca Theologia' e 'Philosophia Perennis': due temi del Rinascimento italiano e la loro fortuna," in *Il pensiero italiano del Rinascimento e il tempo nostro* (Florence, 1970), 211-236.

At a time when the diversity and the incompatibilities of the classical heritage became obvious and the wars of religion shook dogmas and created new ones, it is not surprising that sixteenth-century authors based great hopes on Ciceronian rational compromise, that is, on probabilistic scepticism and eclecticism. In the Reformation world, in particular starting with Melanchthon, this combination was revitalized. In fact, Cicero's logic and epistemology were associated with another ancient authority, Proclus, who had been well known in the German lands already in the late Middle Ages.[10] His thought derived most directly from Plotinus and subsequent neo-Platonists. In the late Middle Ages his *Elementatio theologica* was in fact among the most widely read neo-Platonic texts, both in the Christian and in the Muslim worlds. Proclus' priority had been the conciliation between Plato and Aristotle and the construction of a unity of philosophies and of knowledge. In his theory, universals were depicted as a combination of Plato's ideas and Aristotle's universals, both as the product of intuition and as the result of deduction. For the first principles, Proclus uses two technical terms from the Stoic vocabulary, *koinai ennoiai* and *axioms*. The metaphysical concept of axiom appears in his influential *Commentary to the first book of Euclid's Elements*. Two of the main tenets presented in this book are particularly relevant for our current purposes: the theory of the natural light of reason, according to which the faculty of counting is at the very origin of human knowledge and participates in God's knowledge and his creation of the world; and the theory of a single discipline, based on common axioms, at the root of both arithmetic and geometry. In the sixteenth-century this discipline would come to be called *mathesis universalis*.

Given that in the following sections we shall discuss Euclid's axioms, let us start by looking closely at Euclid's axioms or common notions according to Proclus' interpretation:[11]

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.

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[10] Proclus, who flourished in the fifth century A.D. at the Academy in Athens, was an extremely productive author, contributing forcefully to the spreading of neo-Platonic philosophy.

[11] We quote the standard English translation, Proclus, *A Commentary on the First Book of Euclid's Elements*, transl. intr. and notes by G. R. Morrow (Princeton, 1970), 151-52.

3. If equals be subtracted from equals, the remainders are equal.
4. The whole is larger than the part.
5. Things which coincide with one another are equal to one another.

Proclus remarks:

> This, then, we must accept in advance as the criterion of the peculiar character of axioms and understand that they all belong to the common genus of mathematics. For each of them is true not only of magnitudes, but also of numbers, of motions and of times. This is necessarily so. For equal and unequal, whole and part, greater and less, are common characters of both discrete and continuous magnitudes. (…)
> Furthermore, there is no need to reduce them to the lowest possible number, as Heron does when he proposes three only.

Two important historians of the Euclidean text, Tannery and Heath, were in fact convinced that the last two axioms were introduced by Proclus himself.

Further on, Proclus criticizes Apollonius for having tried to provide a proof for them. The ambiguity of the attribute 'common' was fully exploited by Proclus as well as by his Renaissance readers: notions are 'common' not only because people consider them to be basic, acceptable principles, but also because Proclus developed a theory about a fundamental mathematics, starting with a minimum of axioms, to which each discipline—geometry, on the one hand, and arithmetic, on the other—subsequently adds its own specific axioms.

This dialectical movement had an impact on sixteenth-century mathematicians. Both Agricola and Melanchthon associated the new dialectic with a reinforcement of mathematical education. Mathematicians, whose social status had gained in importance because of the increased teaching of mathematics in the reformed colleges, found themselves in the position of being able to promote an intense debate on the logical status of their discipline. Followers of the Ciceronian tradition based themselves on the theory of enthymemes in order to valorise Greek geometrical analysis and algebra.[12] In so doing they found a new coherence

[12] The classical work on Greek geometrical analysis as an analytical corpus, a logical procedure and a 'toolbox' for problem-solving is M.S. Mahoney, "Another Look at Greek Geometrical Analysis," *Archive for the History of Exact Sciences,* 5 (1968), 318-348.

between their mathematical activity, on the one hand, and, on the other, the logical *habitus* deriving from their own juridical education and that of members of their circle, their readers and patrons. Enthymemes allowed them to treat in a uniform way the scientific logic of mathematics, of the natural sciences, and of juridical arguments.

Traditionally, algebraists had been masters of the *arte della cosa e del censo*, the "art of the thing and of wealth," a special set of techniques connected to commercial arithmetic. According to the new dialectical encyclopaedia, they were found more frequently among the masters of arts, the promoters of a new role of mathematics in university education. From the world of merchants they moved to the *studia*, relegating to the background the traditional connections with trade, while stressing its potentialities in other mathematical or mathematizable fields. Furthermore, they established a new genealogy for algebra, which they retraced to the Alexandrian world—a move that enabled them to reject the Arabic tradition and to legitimate the renewed foundation of algebra within French humanism.[13]

As a consequence, between 1554, the date of publication of *L'Algèbre* by Jacques Peletier du Mans, and 1628, the date of composition of Descartes' *Regulae*, algebraic questions dealt increasingly less with the division of goods, or with alloys and interests, and became instead the art of solving problems of equality between numbers and of proportion between magnitudes. More precisely, according to Descartes, any scientific question well formulated had the shape of an equation.[14] The difficulty to be solved thus turned into the theoretical difficulty of how to put a given problem into an equation. Algebra thereby became more ambitions with respect to the possibility of formulating solvable problems, but also more restrictive with regard to what should be accepted as a well-formed problem, to the point of excluding problems in both natural sciences and in mathematics. This was the highpoint of rhetoric as a structuring element in algebra.

[13] G. Cifoletti , "The Creation of the History of Algebra in the Sixteenth Century," in C. Goldstein, J. Gray, J. Ritter (eds.), *L'Europe mathématique* (Paris, 1996), pp.121-141.

[14] G. Cifoletti, "Quaestio sive aequatio, la nozione di problema proposta nelle *Regulae*," *Da Democrito a Collingwood. Studi di storia della filosofia*, ed. A. Ingegno (Florence, 1991), 43-79. The appendix to my *Mathematics and Rhetoric* contains an English version of this.

From that moment onwards, in keeping with Descartes' program, algebra itself became the basis of a new scientific rhetoric.

In these pages, my main focus will be on the logic (or, more precisely, dialectic) used in algebra and in geometry by some algebraic authors of the sixteenth century. In fact most of the discussion will be about the use of two concepts borrowed from Stoic logic, namely 'common notions' and 'enthymemes'. I shall argue that the reform of logic usually ascribed to Peter Ramus was in fact much larger in scope, involved several other authors and was directly associated with major algebraic changes in sixteenth-century France. This process opened the path to the Cartesian conception of equations as well-posed questions.

Peletier's Version of Euclid's Elements

In 1554, Jacques Peletier du Mans published his famous *L'Algèbre*, which he later translated into Latin and revised in *De occulta parte numerorum* in 1560. Between these two publications, he worked on Euclid's *Elements*, of which he published his own version in Lyon in 1557. It can be shown that he saw[15] his work on algebra as connected to his interpretation of Euclid. Take, for example, the differences—beyond mere formulations—between the two editions of his algebra. Admittedly, they are few in number, but they include demonstrations based on Euclid's *Elements*. Furthermore, his program on a new algebra was intrinsically connected to his rhetorical agenda.[16] Using the untranslatable phrase *l'algèbre apprend à discourir*, Peletier felt that "algebra teaches how to talk and think and to look for all the points necessary to solve a difficulty."[17] Appropriately modified by the new art of thinking (dialectic), algebra itself could become an art of thinking in mathematics.

Here we shall look at Peletier's view of the connections between algebra and Euclidean geometry. First, we shall explore the way in which Peletier deals with Euclid's *Elements* as the main model of mathematical demonstration. Most mathematical authors of

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[15] Jacques Peletier du Mans *In Euclidis Elementa Geometrica Demonstrationum Libri sex* (Lyon, 1577).

[16] See in particular Cifoletti, "La question de l'algèbre."

[17] Peletier du Mans, *L'Algèbre*, 1: "L'algèbre apprend à discourir, et à chercher tous les points nécessaires pour résoudre une difficulté."

the time believed that the extant text of the *Elements* consisted of Euclidean statements and posterior proofs. But an edition of the *Elements* had been a project for many generations of authors, which had become even more urgent with the diffusion of the printing press.[18] The first printed edition (Venice, 1484) was a Latin translation made in the thirteenth century by Campanus of Novara, which was largely based on the Latin translations of Arabic traditions. In 1505 Bartolomeo Zamberti published in Venice a Latin translation from the Greek, in which he criticised the previous publication and added many proofs attributed to Theon. In 1509 Pacioli published his Latin translation of Euclid's *Elements*, based on Campanus and including corrections and annotations. In 1516 Lefèvre d'Etaples (Faber Stapulensis) edited a Parisian edition, which was published by Henri Etienne. This edition had a conciliatory role, because it contained Campanus' and Zamberti's commentaries.[19] It took Zamberti's view seriously, according to whom Theon was responsible for most of the proofs, while Campanus had added some others, with mistakes. In the preface Lefèvre d'Etaples mentions Campanus, Giraldus Odonis and Nicholas Cusanus. The Franciscan Giraldus Odonis was the thirteenth-century 'realist' who introduced the theory of *intentiones*. The name of Nicholas Cusanus (1401-1464), in turn, seems to stand for the mathematics of the infinite as well as for mathematical humanism as the philological project of reconstructing the classical heritage. Lefèvre d'Etaples himself was a key humanist; educated in Italy in a neo-Platonic environment, he was the author of several mathematical editions. It was his intention to provide a historically sound interpretation of the *Elements* by reconstructing the Platonic context of production of the original work in the complexity of the humanistic Platonic and neo-Platonic revival.[20] This project was shared by a number

[18] See P. Riccardi, *Saggio di una bibliografia euclidea* (Bologna, 1887-1893; repr. Hildesheim, 1974); C. Thomas-Stanford, *Early Editions of Euclid's 'Elements'* (London, 1926); M. Steck and M. Folkerts, *Bibliographia Euclideana: die Geisteslinien der Tradition in den Editionen der 'Elemente' (Stoicheia) des Euklid (um 365-300): Handschriften, Inkunabeln, Frühdrucke (16. Jahrhundert); textkritische Editionen des 17.-20. Jahrhunderts; Editionen der Opera minora (16.-20. Jahrhundert)* (Hildesheim, 1981).

[19] *Geometricorum elementorum libri XV*, ed. J. Faber Stapulensis (Paris, 1516).

[20] Euclid of Alexandria, author of the *Elements*, was often confused with Euclid of Megara, Socrates's disciple and the founder of Stoic dialectic. Both aspects agreed with the sixteenth century's dialectical project.

of Euclid editors of the early sixteenth century, including Pacioli, of which each added some special features. In this case, we must mention that the Stapulensis edition lay particular emphasis on the connection between book II of the *Elements*, concerning the applications of areas, and the arithmetical books. As an example, take the use of the term 'magnitude' as a synonym of general quantity, which was later to be such an important aspect of Viète's algebra, but which does not appear in Viète for the first time. In Campanus' edition of Euclid, the term *megethos*, 'magnitude', as used by Euclid in book V, is translated as 'quantity'. The commentaries included in the Stapulensis edition followed in the same path, connecting the relevant propositions of books II and VI with the arithmetical books.

In 1533 the first edition of the original Greek text was published in Basel by the press of Johann Hervagen. Simon Grynaeus, the editor, presented the *Elements* together with Proclus' *Commentary*, a move that was not without consequences.[21] In his preface, Grynaeus associated Euclid's *Elements* with sixteenth-century concerns with method, referring also to dialectic and Galen.[22]

After Lefèvre d'Etaples, Oronce Finé, the first *lecteur royal ès mathématiques*, published in 1536 his work on Euclid's *Elements*, under the title *In sex priores libros geometricorum elementorum... demonstrationes*. Finé used Zamberti's translation of the propositions, to which he added his demonstrations and commentaries. This publication, in turn, had an impact on the Euclid of Peletier, who may be considered a disciple of Oronce Finé.[23] Peletier edited the first six books using Zamberti's translation.[24] He stresses the 'algebraic' role of book II and gives a technique at least as old as Pythagoras' mathematics, which starts with the division of segments, develops a computation on segments where the product is a plane figure and thereby allows mathematicians to calculate the respective value of figures. It thus provides the basis for the algebraic interpretation of some of Euclid's books, starting with

[21] Simon Grynaeus, *Euclidis Elementorum libri XV ex Theonis coloquiis. In primum ejus librum commentariorum Procli libri IV Graece* (Basel, 1533).

[22] I am preparing a fuller account of this text.

[23] See N.Z. Davis, "Sixteenth-century French Arithmetics on the Business Life," *Journal of the History of Ideas*, 21 (1960), 18-48, at 31.

[24] J. Scheubel's *Euclidis Megarensis, philosophi et mathematici excellentissimi, sex libri priores* (Basel, 1550) must also have been important for Peletier, who had read Scheubel's *Algebrae compendiosa facilisque descriptio* (Paris, 1552).

the geometrical proof of special products. This algebraic role can be seen as the sixteenth-century version of the 'geometric algebra interpretation' provided by some twentieth-century historians.[25]

But how exactly does Peletier navigate between philology and mathematical doctrine? Stapulensis' edition had already established itself when Peletier, fifty years later, took up the project. The *Elements* were still textually unstable with respect to the authenticity of proofs and of books XIV and XV. Peletier assumed the task of providing new proofs, at least for a good part of the text he dealt with, i.e., the first six books[26]. Re-examining Euclidean proofs was not Peletier's key motivation. What he found highly important about the Euclidean text was the concatenation of mathematical statements whose order was a challenge for the mathematical reader. Peletier suggested a few rearrangements in the flow of theorems, problems, lemmas and corollaries. He moreover developed a philosophy of mathematics and a specific vocabulary, which was to have a strong impact.

In the first book Peletier shows his dependence on Proclus' *Commentary* by introducing Euclid's text with a short discussion of principles, that is, of definitions, postulates and common notions.[27] Peletier, summarising Proclus, stresses that both definitions and postulates are such that, once they are proposed, we admit them without effort, while common notions are immediately evident, so that *the person who does not grasp common notions is also lacking common sense*.

Peletier, paraphrasing Proclus, adds philosophical commentaries that had been absent in previous versions of the *Elements*. But the actual principles he uses, the definitions and propositions, are those of the Stapulensis text. Similarly, he gives a list of common notions, which he calls *animi notiones*, 'notions of the mind'. His list, which differs from that accepted today, follows, however, the view of his age: the same list is found in the Stapulensis edition and, under the heading '*communes animi*

[25] On this issue, see in particular M. N. Fried and S.Unguru, *Apollonius of Perga's Conica: Text, Context, Subtext* (Leiden, 2001); A. Bernard, "Ancient Rhetoric and Greek Mathematics: A Response to a Modern Historiographical Dilemma," *Science in Context*, 16 (2003), 391-412.

[26] Only two years later, in 1559, Jean Borrel maintained that the proofs were indeed Euclid's own.

[27] The only available edition of Proclus was Grynaeus', of 1533 (see fn. 20). It was replaced by Franciscus Barocius' version in the Padua edition of 1560.

conceptiones' in the Campanus edition, and as '*communes sententiae*' in both the Theon-Zamberti and the Oronce Finé version. As for Peletier, he gives the following list, to which he adds his own general remarks:

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders will be equal.
4. If equals be added to unequals, the wholes will be unequal.
5. If equals be subtracted from unequals, the remainders will be unequal.
6. The doubles of the same thing are equal to one another.
7. The halves of the same thing are equal to one another.
8. Things which coincide with one another are equal to one another.
9. The whole is greater than the part.
10. Two straight lines do not contain a plane.

Peletier's most important change is that he calls these 'common notions' by the name of '*animi notiones*' and thereby identifies them with Cicero's '*notiones communes*'; in fact, given his dependence on Cicero, he could not have missed this connotation of the term.[28] In other words, though he chose to make use of Campanus' list of axioms, Peletier offered a 'new' interpretation of them in neo-Platonic terms, as is indicated by his references to Proclus' text and the use he made of the Ciceronian term '*notiones*'. It is no coincidence that, slightly later in the century, Proclus' *Commentary* and the status of Euclid's axioms were to become important topics of debate. The rediscovery of Proclus' *Commentary* in fact led to two main philosophical debates in sixteenth-century mathematics, the first pertaining to *mathesis universalis* and the second to *de certitudine mathematicarum*.[29] In fact, only three years after Peletier's work on Euclid's *Elements* had come out, Francesco Barozzi published his edition of Proclus' *Commentary* as well as two 'questions', one concerning *de certitudine* and the other *de medietate*.

But as we shall see in greater detail, 'common notions' or 'axioms' are of great importance not only as the main expres-

[28] The role of Cicero in Peletier's work is discussed in my *Mathematics and Rhetoric*. On Peletier's role in the literary tradition and especially in connection with astronomy, see I. Pantin, *La Poésie du ciel en France dans la seconde moitié du seizième siècle* (Genève, 1995) and eadem, «La représentation des mathématiques chez Jacques Peletier du Mans: Cosmos hiéroglyphique ou ordre rhétorique?» *Rhetorica* 20 (2002), 375-389. On Cicero's philosophy, see James M. May (ed.), *Brill's Companion to Cicero: Oratory and Rhetoric* (Leiden, 2002).

[29] See the classical works by G. Crapulli, *Mathesis universalis* (Rome, 1969) and C. Sasaki, *Descartes' Mathematical Thought* (Dordrecht, 2004).

sion of the 'natural light' of reason, but also because arguments or enthymemes are built on them, presupposing them as basic truths.

After the presentation of the principles and before treating the first problem of book I of the *Elements* (the construction of an equilateral triangle), Peletier inserts a most interesting section on hypotheses, demonstrations, problems and theorems. We give a translation of the entire section on hypotheses because of its interest for our purposes:

> A hypothesis, insofar as it is concerned here, is the underpinning or foundation of reasoning. It is not necessary for it to exist in reality; it is sufficient for it to be probable and for it to follow a rule to avoid absurd implications. And when it turns out that a figure is equal to another or greater than it, and from this an argument is deduced, such equality or inequality forms the hypothesis. And as long as the argument remains intact, it is not permitted to deviate from it.[30]

Hypotheses are thus, in general, not necessarily 'real' (*hanc re existere non est necessarium*), but they should not be absurd nor give rise to absurd implications. The main point is that they are assumed to be true. Peletier then moves on to demonstration:

> Demonstration is called by Dialecticians "Syllogism which makes knowing", that is to say which concludes from demonstrated premises. And this takes its origins from Geometry: actually any proof leading to truth is geometrical. This is why it is rightly said that nobody, who is not familiar with Euclid, can distinguish true from false. If somebody investigates more attentively why in the demonstration of propositions the form of a syllogism does not appear clearly, but only some parts of syllogisms are shown concisely, then that is because it is not dignified, when facing a real situation, to conform strictly to what is taught in the schools, according to the prescribed schemes. For the Orator, when he ascends to the forum, does not explicitly deliver what he has learned from his teacher of rhetoric. Rather, he acts in such a way that, while he does his best to remember the rhetorical precepts, he appears to think about anything except rhetoric. So, in doing geometry, given that we do not look for anything but to reach our goal in a perfect way, we dissimulate entirely the figure of the syllogism. However, if it is required, it can be extracted from the heart of geometrical demonstrations. We cut them

[30] Jacques Peletier du Mans, *In Euclidis Elementa geometrica demonstrationum libri sex* (Lyon, 1557), 12: «Hypothesis, quantum huc attinet, est subiectio, seu fundamentum ratiocinationis. Hanc re existere non est necessarium : probabilem esse satis est :eamque legem duntaxat habet, nequid absurdi importet. Ut quum convenerit Figuram Figurae esse aequalem, aut maiorem, atque ex hoc deducatur argumentatio: ea aequalitas aut inaequalitas, Hypothesis est. A qua discedere, constante argumentatione, non licet.» My translation takes into account the French translation contained in *Les six premiers livres des Eléments géométriques d'Euclide, avec les démonstrations de Jaq. Peletier… traduit en François* (Geneva, 1611).

> out, because repetitions would not only produce boredom, but also obscurity. They can easily be filled in once the conclusion is obtained. The conclusions of demonstrations are problems and theorems.[31]

This section depends directly on Proclus' discussion of these topics in the second part of the prologue of his *Commentary*. The 'dialecticians' Peletier has in mind were medieval authors; but Proclus himself had also been a dialectician, interested in the integration of Stoic logic and the reconciliation of Plato's with Aristotle's philosophy. In the passage just cited, Peletier reveals his strong commitment to Proclus' philosophy and to an interpretation of the *Elements* in which what counts is their logic. But 'logic' must here be understood as the use of cogent arguments in discourse, not scholastic logic, in which a unique structure validates utterances by means of its isomorphism with a unique ontology. With his words about the ability to hide techniques in the fine texture of the discourse, Peletier hints at the debate concerning the identity between mathematical and Aristotelian logic, that is, about the compatibility between Euclidean demonstrations and syllogisms. The difficulty in articulating this identity, or compatibility, had already been clear to Proclus, and sixteenth-century authors were also quite aware of it.[32]

Peletier appears thus as the representative of a particular sixteenth-century view of mathematics: he did not believe that Euclidean demonstrations were syllogisms, at least not in the sense of traditional syllogisms of the first figure. He thought

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[31] Ibid., "Demonstrationem vero appellant Dialectici, Syllogismum qui faciat scire: nempe qui ex probatissimis concludat. Atque haec a geometria ortum habet. Immo omnis quae ad verum perducit probatio, Geometrica est. Ut verissime dictum sit, neminem scire verum a falsum distinguere, cui Euclides non fuerit familiaris. Quod siquis attentius sciscitabitur, quur in Demonstrandis Propositionibus non eluceat forma Syllogismi, sed tantum concise quaedam membra Syllogismorum appareant: is sic habeat, praeter dignitatem esse, quae in scholis docentur, ea quum in rem praesentem ventum sit, ex praescriptis formulis observare. Neque enim Orator, quum ad forum accedit, quae a Rhetore excepit dictate, in digitis collocate: immo id agit, ut quum praeceptorum maxime meminit, nihil minus quam Rhetoricen cogitare videatur. Sic in opera Geometrico, quum nil aliud spectemus quam ut scopum exquisite assequamur. Syllogismi figuram omnino dissimulamus. Quae tamen si exigatur, e probationibus Geometricis ad vivum exprimitur. Sed nos ea resecamus, quae repetita non modo taedium, sed etiam obscuritatem parerent. Quod inter Demonstrationes facile perspicient qui iudicio praediti erunt. Demontrationum autem conclusiones, sunt Problemata et Theoremata."

[32] Incidentally, their arguments have some affinity with those of Orna Harari, *Knowledge and Demonstration: Aristotle's Posterior Analytics* (Dordrecht, 2005).

that the Euclidean demonstrative system needed to be revised by re-ordering several theorems, so as to show the mutual dependence of propositions more clearly and to follow more directly the natural flow of human understanding. But he also believed that mathematics was the primary science and that "nobody can distinguish true from false without being familiar with Euclid." The value of mathematics was intrinsic, since "any proof leading to truth is geometrical." The basis for this belief is explained in the preface addressed to Charles de Lorraine. Peletier there begins by saying that the main reason to recommend mathematics, above and beyond its utility and value, is the fact that while other arts consist of probable opinions, mathematics consists of true statements, and that nothing counts in it but order and *ratio*. He concludes with the claim that there is no point in ascribing the origin of geometry to a particular people, be they Egyptians, Chaldeans or Phoenicians, because geometry is in fact a sort of theory of the world and connected to creation itself. And

> in the same way in which the constitution of the world has been inscribed in the divine Mind from eternity, so the disciplines are a sort of celestial seeds implanted in us, which yield fruit according to the care we put in making our portion grow.[33]

Peletier expressed this belief many times in terms of "flames situated in the human mind," i.e., in terms of the light of nature within the human mind which, as Melanchthon had explained, consisted in the faculty of counting and in basic notions [34] What was usually called mathematics is a more articulate product of the same faculty: given that Euclid's *Elements* are the core of the classical mathematical corpus, he is here taken as a synonym of mathematics as a whole. But we have also seen that the basic notions are common to everybody and also to all mathematical sciences.

Now, Euclid's *Elements* contains more than one level. In modern terms, there is, on the one hand, the language of its theorems and theory and of its concatenation of theorems with the

[33] J. Peletier *In Euclidis Elementa* (end of the *Praefatio*): "Ut in mente divina ab aeterno infixam fuisse Mundi constitutionem: sic disciplinas, caelestia quadam semina esse: quae in nobis insita, et pro rata cuiusque portione exculta, fructum edunt."

[34] See Philip Melanchthon, *Liber de anima* in Philippi Melanchthonis Opera ed. C. G. Bretschneider (Braunschweig, 1846), XIII: 138, "De potentia rationali seu mente." In fact, for Melanchthon, the *lumen naturae* proves the existence of the mind qua *architectatrix sapiens*.

rules of deduction, the logic of which may justly be subjected to scrutiny. On the other hand, there exists also a meta-language in which the theory is presented and which follows its own logic. Peletier is sensitive to the existence of these two levels of Euclidean logic (leaving aside here the further element of the logic of discovery). It is with respect to these levels that he brings up the *topos* of the difference between rhetorical rules and actual discourse, applying it precisely to the non-reducibility of Euclidean demonstrations to Aristotelian syllogisms. He addresses the question of how demonstrations which do not follow the procedures that guarantee valid consequences, notably first figure syollogisms, can lead to the truth. He considers that it is important to suppose that Euclid tried to organise his subject-matter according to some well-defined criteria, irrespective of the degree to which his debt to the general rules of rhetoric or dialectic are rendered manifest. The problem of Euclidean logic is thus for Peletier a problem of structure and presentation, or 'disposition'. This view of things is consistent with his statement, in *L'Algèbre*, where he characterizes the work of the author of mathematical texts as 'disposition'.

Let us see an example of how this idea could work in mathematical practice. It is taken from Euclid's *Elements*, book I, proposition 43. In Peletier's terms, this proposition states: "Of two parallelograms about the diameter of a greater parallelogram, the supplements are equal."[35]

But Peletier has changed the original wording. He claims to have changed the *structura* of the proposition. This term, *structura*, does not concern the proof itself, but its construction. Peletier starts from a definition of 'parallelogram about the diameter' and of 'supplements', then moves to the construction of two parallelograms with diameters on the same line and with a common vertex: they will, he says, have as their diameter the diameter of a parallelogram composed of the two (see fig. 1).

[35] Peletier du Mans, *In Euclidis Elementa*, 40: «Duorum Parallelogrammorum circa Dimetientem maioris Parallelogrammi consistentium, Supplementa sunt æqualia.»

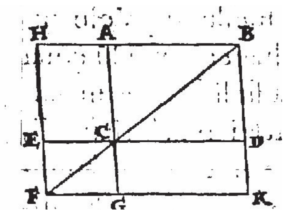

Figure 1. "Two parallelograms about the diameter of a greater parallelogram," from Peletier's Euclid edition, p. 41.

Campanus, Zamberti, and the very construction which has come down to us as 'Euclidean'—even that presented in Heath's version—do not provide any definitions, but start right away with the construction of the larger parallelogram. In its main steps, Peletier's proof conforms to that provided by the other authors and that presented in Heath's version. In all cases, the proof rests on proposition 34: "In any parallelogram, the opposite sides and angles are equal to one another, and the diameter bisects the parallelogram." The next step is consider the fact that on each side of the diameter there is a larger triangle made of two triangles and a supplement. As Campanus and Zamberti had pointed out in their version of the proof, we make use of the third *animi conceptio* or *communis sententia* and thereby conclude that the result of the subtractions are equal.

But Peletier, as we have mentioned, has changed the construction. He claims that he has made the change in the "structure," not because he seeks novelty, "but in order to make evident all the business of the supplements and of the parallelogram as a whole."[36] His goal is to stress the importance of the proposition, and so he goes on:

> For, almost nowhere in the whole geometry is there a more fruitful *figuratio* than this *gnomica,* i.e. that is, one in which parallelogram and a gnomon are fused into one. In this place, if we take ABCD as the parallelogram, the figure HFCD, which together with ABCD completes the parallelogram BHFK, is called *gnoma* or *gnomon*.[37]

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[36] Ibid, 41: "In hac Propositione demonstranda, structuram ab aliis aliquantum variavi, non novitatis studio, sed ut totum negotium Supplementorum et integri Parallelogrammi evidentius exponerem."

[37] Ibid.: "Vix enim usquam in toto opere Geometrico occurrit Figuratio magis fœcunda quam hıc Gnomica : hoc est, quı uno parallelogrammo et gnoma conflatur. Ut hoc loco, si ABCD parallelogrammum sumamus, Figura illa HFCD, quı cum

Peletier insists on the fact that this 'locus' is the 'most appropriate' one to explain the gnomon, "even though Euclid has deferred" this explanation "to the second book" of the *Elements*.[38] Peletier then adds: "I call this figure 'mystical', because from it, as from an extremely rich depository, arise innumerable demonstrations."[39] We shall not here embark on a comment on the strong term 'mystical'. Peletier may have used it very consciously with regard to that part of Euclid that was attributed to the Pythagoreans, namely with regard to the so-called theory of application of areas. The question here concerns rather the nature of the most relevant applications of the gnomon. The answer to this is found in book II, in the theorems which have appeared to some historians and also to some sixteenth-century mathematicians as algebra in geometrical disguise.

In book II, Peletier deals with the proposition that had been mentioned by Pacioli as the basis for the solution for cases of second degree equations (see fig. 2). Incidentally, Peletier's interest in this demonstration is also evident from his *L'Algèbre*.

With his treatment of the gnomon, Peletier exercised some influence, not least on the authoritative Christoph Clavius, "the Euclid of the sixteenth century," who included it in his Euclid commentary. Already in his first edition of 1574, Clavius expanded the relevant section to follow Peletier's 'structure', and he included the definition of 'gnomon' among the definitions at the beginning of book I. Therefore, Clavius did not stress the importance of the special construction of the gnomon in connection with I, 43. Instead, he pointed out that Proclus had developed another demonstration. Peletier's treatment, together with an explicit reference to him, can be found at the beginning of book II of the *Elements*. Clavius, well known for his striving for a rigorous proof structure in Euclid—he gives a syllogistic demonstration of the first proposition of book I of the *Elements*—takes up Peletier's lesson. He states that in book II, Euclid gives a geometrical interpretation of arithmetical operations, and what matters in particular is the operation of the product. He introdu-

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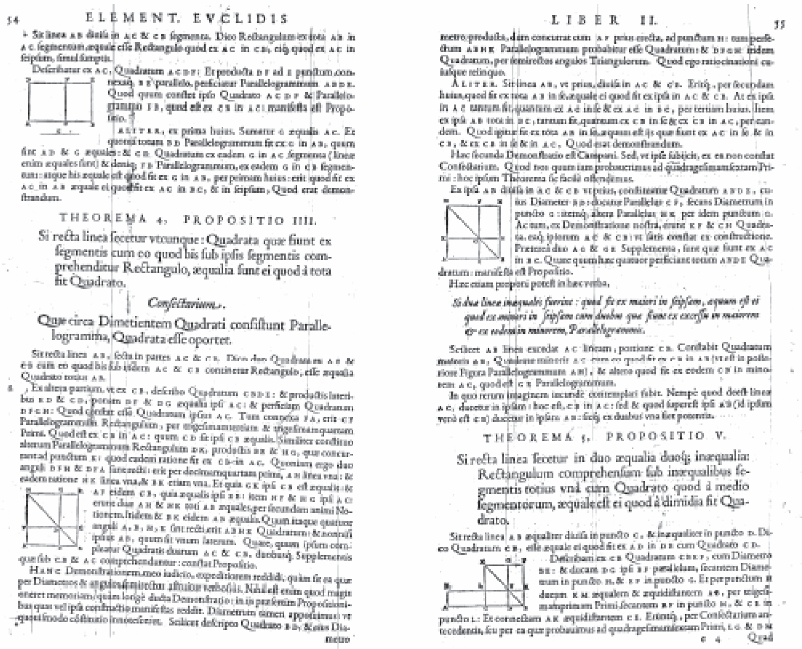
ABCD perficit totum BHFK Parallelogrammum, Gnoma seu Gnomon vocatur."

38 Ibid. : "Nam hic Gnomonis explicandi locus est maxime oportunus: licet Euclides ad secundum librum distulerit."

39 Ibid. : "Hanc ego Figuram mysticam soleo vocare: Ex ea enim, veut ex locupletissimo promptuario, innumerabiles exeunt Demonstrationes."



54 ELEMENT. EVCLIDIS

THEOREMA 4. PROPOSITIO IIII.

Si recta linea secetur vtcunque: Quadrata quæ fiunt ex segmentis cum eo quod bis sub ipsis segmentis comprehenditur Rectangulo, æqualia sunt ei quod à tota fit Quadrato.

Consectarium.

Quæ circa Dimetientem Quadrati consistunt Parallelogramma, Quadrata esse oportet.

LIBER II. 55

THEOREMA 5. PROPOSITIO V.

Si recta linea secetur in duo æqualia duoq; inæqualia: Rectangulum comprehensum sub inæqualibus segmentis totius vnà cum Quadrato quod à medio segmentorum, æquale est ei quod à dimidia fit Quadrato.

Figure 2. Some of Peletier's solution for cases of second degree equations

ces an explicit comparison between the two species of quantities, developing further what Pedro Nunes, another sixteenth-century author certainly read by Clavius, had written in 1567.

This passage shows Clavius' dependence on Peletier, even though he does not invert Euclid's 'ordo'. In book II, Clavius chooses to treat the theorem of book I on parallelograms, explaining that his demonstration follows Peletier's, "which is very clear."[40]

In this section we have tried to understand what moved Peletier, after publishing his algebra, to write a book to Euclid's *Elements*. There are certainly several reasons for this, and they are likely to have something to do with John Dee's famous and greatly successful lectures delivered in Paris in 1550. Two main elements have emerged during our discussion: the Proclean idea

[40] C. Clavius, *Opera Mathematica* (Mainz, 1612), 28-9. I expect to publish shortly, with Samuel Gessner, *Algebra in Euclid*, a detailed comparison between the various interpretations of book II of the *Elements* by sixteenth-century algebraist such as Nunes, Tartaglia and Clavius.

of a mathematical 'natural light', made explicit through the 'common notions', and the idea of a science common to both arithmetic and geometry, which was being identified with algebra. These two ideas motivated Peletier throughout his career and encouraged him to study those features of mathematics that make it different from any other science and make of it a privileged way to superior knowledge.

Peletier shared these views with Petrus Ramus, his contemporary, who had published a work on Euclid's *Elements* in Paris already in 1549.[41] This text may seem disappointing to the modern reader, for it is a list of all Euclidean definitions, postulates, axioms and propositions. It would be important to understand in what context this kind of text could be used. It certainly was conceived as a teaching tool, but was it actually used in any college or even at the Collège Royal? In any case, it is noteworthy that the dedicatory letter to Charles de Lorraine—the same patron as Peletier's—contains a sentence that points in the same direction as Peletier's preface, by declaring that, according to Proclus, for Pythagoras and Plato, these 'notions' were innate and should for this reason be called by the name of 'mathesis', or, as it were, 'reminiscence' or 'remembrance'.[42]

But let us take a more systematic look at Ramus' texts. Particularly his book on dialectic places some of Peletier's mathematical ideas in a broader context. Indeed, as we shall see, Peletier and Ramus both worked on the logic of Euclid's *Elements* and both considered the popular project of trying to translate Euclid's demonstrations into Aristotelian 'scientific' arguments (first figure syllogisms) to be useless.

Ramus' Dialectique

Ramus' *Dialectique* of 1555 had a great impact on its readers. It also presented several theses that are very important for our current purposes. In the dedicatory letter, which is once more addressed to the Cardinal Charles of Lorraine, Ramus defines

[41] Petrus Ramus, *Euclidis Elementa mathematica* (Paris, 1549).

[42] Ibid., page 2 of the dedicatory letter: "(Pythagoras and Plato) in anima tam excellentes notitias ab intelligentiae primae ingenitis et aeternis exemplis insitas, et ingeneratas crediderunt: eique propterea nomen ipsum matheseos, quasi reminiscentiae, recordationisque (ut Proclus author est) affinxerunt."

'dialectic' as the "art général pour inventer et juger de toute chose." The issue is to replace the two Aristotelian disciplines, logic and dialectic, with a single art of thinking. This is made clear in the incipit of the first book, where we read:

> La Dialectique est l'art de bien disputer. Et en même sens elle est nommée Logique, car ces deux noms sont dérivés de *logos*, c'est-à-dire raison. (…) Ainsi devons-nous apprendre la Dialectique pour bien disputer à cause qu'elle nous déclare la vérité, et par conséquent la fausseté de toute raison, soit nécessaire, dont est la science, soit contingente, c'est-à-dire qui peut être et ne pas être, dont est l'opinion.[43]

This distinction between science and opinion is as important for Ramus as for any follower of classical philosophy, but

> Platon et Aristote ont ouvertement déclaré que l'homme était capable de science comme dans les choses comprises par les disciplines, et d'opinions comme dans les affaires infinies qui surviennent d'heure en heure. Mais à cause de ces deux espèces, Aristote a voulu faire deux Logiques, l'une pour la science, l'autre pour l'opinion, en quoi (sauf l'honneur d'un si grand maître) il a très grandement erré. Car bien que les choses connues soient les unes nécessaires et scientifiques, les autres contingentes et objet d'opinion, toutefois il est vrai qu'ainsi que la vue est commune à voir toutes les couleurs (…) ainsi l'art de connaître, c'est-à-dire Dialectique ou Logique, est une seule et même doctrine pour apercevoir toutes choses. (…) Par conséquent, nous dirons que la Dialectique est l'art de bien disputer et raisonner de quelque chose que ce soit, tout ainsi que la grammaire est l'art de bien parler de tout ce qui pourrait offrir et proposer.[44]

This statement is very rich in consequences: given that the same logic is used for 'science' and for 'opinion', there is no way to protect 'science' *logically* from the irruption of mere 'opinions'. Aristotle's concern with formal exactness was founded on the strong presupposition of a continuity between thought and things, which is not shared by Ramus, who had inherited Cicero's Stoic view of the fragile nature of the premises: if the latter happen to be false, the whole art of reasoning will be unable to save a single conclusion.[45] However, within a science, it is well possible to follow Aristotle—as the Stoics did, too—and to distinguish between necessary and probable statements, between axioms and propositions.

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[43] Pierre de la Ramée, *Dialectique* (Paris, 1555). Texte modernisé par Nelly Bruyère (Paris, 1996), 16.

[44] Ibid., 18.

[45] We should remember that the logic of the Stoics stood at the cradle of the *modus ponens*, a syllogism which makes a conclusion valid if there is a connection between the premise and the conclusion and the premise is valid.

Further on in Ramus' text, we find the definition of dialectic as the art of 'reasoning well' given as the case of a "universally reciprocal axiom":

> La Grammaire est l'art de bien parler, la Rhétorique de bien dire, la Dialectique de bien raisonner, l'Arithmétique de bien compter, la Géométrie de bien mesurer. Ces proposition sont universelles premièrement, c'est-à-dire des axiomes réciproques.[46]

So, although he maintained the distinction between scientific judgements and opinions, Ramus minimized the other distinction, that between necessary and probable judgements. The same art of reasoning now includes both kinds of judgments without a hierarchy of validity.

According to Aristotle, scientific judgements are syllogisms. In fact, mathematics appears in the dialectic exclusively in connection with syllogisms. But then, what is a syllogism? Ramus' answer is that it consists of counting:

> Aristote prend souvent logisme pour syllogisme, comme au premier du *Syllogisme* [= Prior Analytics] deuxième de la *Démonstration* [= Second Analytics], sixième des *Topiques*, deuxième de la *Physique*, première et septième de la *Philosophie* [= Metaphysics]. Et tous ces deux mots signifient proprement compte et dénombrement. En ce sens, l'arithmétique est nommée logistique. Et il semble que ces vocables soient traduits de mathématiques en dialectique car comme le bon compteur, en ajoutant et déduisant, voit certainement en la clôture du compte le reliquat, ainsi les dialecticiens en ajoutant la proposition et en déduisant l'assomption, voient en la conclusion la vérité ou fausseté de la question.[47]

Ramus explains that, in a logical text, dealing with the notion of enthymeme is not necessary, because it suffices to talk about syllogisms. All reasoning, all judgement consists of syllogisms. However, in most cases, the actual expression of reasoning is a syllogism missing one term. It should, therefore, properly (and by the authority of Aristotle and others) be called 'enthymeme'. This transition occurs in Ramus' "Péroraison du premier jugement." There, Ramus deals with the first judgement, that is, with "l'énonciation scientifique immédiate," which can be an axiom or a thesis. In both cases it will respect the three rules that Ramus takes from the *Posterior Analytics* and identifies with his method: "Toute énonciation marquée de ces trois marques du tout, par soi, universel premièrement est un vrai principe

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[46] de la Ramée, *Dialectique*, 85.
[47] Ibid., 59.

d'art et de science et première cause."[48] Ramus then offers his definitions of 'axiom':

> L'axiome est un principe représentant son intelligence aussitôt qu'il est énoncé, comme:
> Le tout est plus que sa partie.
> Deux plus deux font quatre.
> Et toutes ces intelligences qui sont bien claires à notre première et naturelle raison sans aucune observation ni expérience d'aucun sens, voire sans aucune doctrine antécédente.[49]

This is followed by a definition of 'thesis':

> La thèse est un principe qui n'est pas aussitôt aperçu mais qui requiert le sens et l'expérience de quelques exemples familiers… D'une telle sorte est la plus grande partie des arts et disciplines qui ont été connus par expérience et observation des choses sensibles et singulières …aussi dans la *Démonstration* <Aristote> enseigne que l'esprit, bien qu'il n'apporte pas avec soi en nos corps la connaissance des choses, a néanmoins la puissance naturelle de les connaître….Ainsi donc si quelque obscurité se trouve dans les principes, alors ils sont appelés thèses par Aristote et pour leur éclaircissement, nous avons recours aux exemples par lesquels ils ont été aperçus.[50]

His intention of connecting this logic to the natural language is once more outspoken:

> Et c'est pourquoi, en tout l'art de la Dialectique et toutes ses règles que nous suivons, nous n'avons tenu ni ne tiendrons d'autre voie que de déclaration que celle d'exemples les plus insignes et familiers que nous avons pu choisir par une longue étude et recherché dans l'usage naturel et la vraie pratique de la raison.[51]

So, going back to the notion of thesis,

> Mais telles sont les énonciations nécessairement vraies, et vraies sans moyen <terme>, qui est la matière de l'art que nous avons allégué en notre préface. Par conséquent, ces énonciations scientifiques sans moyen <terme> relèvent de ce premier jugement, comme aussi les énonciations contingentes.

With apparent ease, Ramus has connected the two sorts of statements, 'scientific' and 'contingent'. He points to the *quaestio* as to the way to distinguish between them:

> Or si l'énonciation, soit nécessaire, soit contingente, n'est point manifeste mais douteuse et incertaine, comme le sont les infinis débats et procès entre les hommes, elle est convertie en question et alors l'antécédent est appelé par Aristote au premier du *Syllogisme*, terme mineur, le conséquent, terme

48 Ibid., 57.
49 Ibid., 58.
50 Ibid.
51 Ibid.

> majeur; et le jugement de celle-ci appartient au syllogisme duquel il nous faut donc parler.[52]

This role of the *quaestio* is another typical innovation of Valla's dialectic that is inserted here, together with the value attributed, because of its frequent use, to the abbreviated syllogism (the 'enthymeme' of previous contexts):

> Or l'usage du syllogisme entier est très rare, car souvent, et presque toujours chez les poètes, les orateurs, philosophes, et tous les auteurs suivant l'usage naturel, encore qu'ils traitent des questions syllogistiques, néanmoins quelque partie du syllogisme est délaissée, et tel syllogisme imparfait est nommé par Aristote enthymème au deuxième du *Syllogisme*.
>
> L'esprit de l'homme parfois est content de la seule proposition, parfois de l'assomption, parfois il conçoit plutôt la conclusion afin qu'elle se puisse dire et exprimer; néanmoins, en examinant ce jugement syllogistique, il faut remplir les parties qui sont seulement entendues et achever le syllogisme.[53]

This is the key passage for our argument: here, Ramus states clearly that in most cases syllogisms are abbreviated syllogisms, or enthymemes, and he therefore concludes that a special treatment of enthymemes is no longer necessary. Precisely because it is used to define general syllogisms, the original notion of enthymeme simply disappears.

This chain of reasoning may constitute a possible source for Peletier's preface to Euclid. Writing only two years later, in a cultural context not different from Ramus', Peletier shared many aspects of his scientific program. Peletier's idea that Euclid's *Elements* was written in rhetorical or abbreviated syllogisms which can easily be completed may be said to appear in Ramus at the level of dialectical theory.

Ramus' Algebra

In 1560, that is, three years after Peletier's *Euclid* and six after his own *Dialectique*, Ramus published a treatise on algebra. This allows us to see whether he finds a concrete use of dialectic in mathematics, and whether it has a role in shaping the new algebra. Published anonymously, and thus, for once, without the patronage of Cardinal Charles of Lorraine, Ramus' *Algebra*, following Cicero's rhetorical rule, is particularly 'brief and clear'. It is also quite

[52] Ibid.
[53] Ibid.

dry: there is no place for the typical Ramist eloquence. The 18 folios are divided into two books, entitled *numeratio simplex* and *numeratio comparata in aequatione*. *Numeratio simplex* is devoted to the explanation of the four operations on figurate numbers, usually called cossic numbers in the sixteenth century and nowadays known as monomials. *Numeratio comparata in aequatione* is devoted to the solution of first and second order equations.

In the chapter devoted to second degree equations, Ramus provides a demonstration of the solutions of the three cases, relying on book II of the *Elements* like Luca Pacioli shortly before him. The Euclidean propositions mentioned are the 4^{th}, 6^{th} and 5^{th}, respectively for the first, second and third *canon*. Ramus gives very short proofs without figures. Let us look at the first one, as an example:

> *Second equation, first canon.*
> The first canon is deduced from II, 4 of the Elements. For, 1 q + 8 l is put equal to the gnomon, i.e. 65, and will be equal to it. But q is the square of the gnomon: therefore 8 l are equal to two planes, and 4 l is any of the equal planes, the length of which 1 l is the side of the square already determined, whereas the breadth 4 is the side of the other square. Therefore 16 will be the last square itself, and added to the gnomon 65, will complete the whole square 81; if we take away from its side 9 the side 4 of the second square, it will remain 5, side of the first.[54]

By omitting the figure, Ramus has simplified the typographical process. But the computation on figures is implied from the outset, because Ramus is clearly making use of the application of areas. The equation is produced by 'putting equal.' To what extent is this innovative? Conceptually, Ramus has absorbed the connection between algebraic quantities and figures so thoroughly that figures have become dispensable for him. A further typographical simplification he introduces is the use of letters of the alphabet instead of the traditional special symbols for 'figurate numbers' designating the powers of the unknown.

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[54] *Algebra* f. 14 : «Primus Canon deduciturque e 4.2 Elem. Nam 1 q—8 l aequatur gnomoni, ideoque & 65, eidem aequantur : at 1 q est gnomonis quadratus: ergo 8 L sunt duo plani aequales & 4 l planus alter aequalium planorum , cuius longitudo 1 l est latus quadrati jam positi: latitudo autem 4, est latus reliqui quadrati. Quare 16 erit quadratus ipse reliquus, additusque gnomoni 65, explebit totum quadratum 81, a cuius latere 9 si tollatur 4 latus secundi quadrati, restabit 5 latus primi. Sed Canonis exempla pleniora statuantur.»

Ramus and Peletier on Mathematics, and their Opponents

Peletier and Ramus take a shared view on mathematics and in particular on mathematical demonstration. As for mathematics, it proceeds, for both authors, from a basic human faculty, called 'natural light' or *notions communes* by Peletier and 'natural light' or *mathesis* by Ramus. In agreement with Lorenzo Valla's logic, the particular evidence of mathematical statements and the particular strength of mathematical arguments rest for both authors on the special relation of this 'natural light' to truth and on its status as the basis of human knowledge. This basis is not only probable but certain. Human participation in the knowledge of the world is taken to have been present since its very creation. Furthermore, demonstration viewed as the judgment of a thesis based on axioms and experience. Mathematical demonstrations, like most demonstrations, are syllogisms that do not appear as such, that is to say, abbreviated syllogisms including only the axiom useful for the conclusion, without a middle term. This way of reasoning is the most natural, and corresponds to the reasoning expressed in spoken language. From this point of view, to look for a syllogistic form of mathematical demonstrations is at best an innocent pastime and at worst a way to conceal the natural understanding of the most natural kind of knowledge.

Only a few years later, their project was considered important by some contemporary specialists of mathematics and of logic such as Konrad Dasypodius and Jakob Schegk. The former, who was a disciple of Johann Sturm, continued to transmit Agricola's dialectic, that Northern version of Lorenzo Valla's dialectic, in the Strasburg Gymnasium. Like Ramus, Dasypodius adhered to a notion of *mathesis universalis*. Among his several publications, there is one, co-authored with Christian Herlinus and entitled *Analyseis geometricae sex librorum Euclidis primi et quinti* (1566), which offers a reduction of some of the books of the *Elements* to syllogisms. To name a negative reaction, too, in 1570, Jakob Schegk, doctor of medicine and professor of philosophy at the University of Tübingen from 1543 to 1587, wrote a theoretical pamphlet against Ramus' dialectic.[55] In this text, Peletier is also

[55] Jakob Schegk *Hyperaspites responsi ad quatuor epistolas P. Rami contra se aeditas* (Tübingen, 1566). Jakob Schegk was the author of a number of commentaries on Aristotle, but especially of the great *De demonstratione libri XV* (Basel, 1564).

mentioned and explicitly criticised because of his geometry; in particular, he is accused of reducing mathematics to a language, in order to empty it of its object. But Schegk's main target was Ramus' dialectic which was charged of neglecting the role of 'analytic', the theory of the syllogisms leading to true conclusions. Schegk restates the strict Aristotelian doctrine according to which dialectic is the logic of enthymemes, leading to probable conclusions.[56] In fact, two interpretations of Aristotle confront each other here, of which each in turn includes a different reading of Plato. While Ramus seems to depend mostly on Proclus, Schegk, the theoretical physician, has mostly Galen in mind.[57] The true Aristotelian method was, for Schegk, only apodictic, so that geometrical demonstrations should conform to it, while for Ramus, it was contained in the three rules of method he had extrapolated from the *Posterior Analytics*. In the fifties, Peletier had started to propose an interpretation of Euclid in which conformity to the apodictic standard was not crucial for truth. For both Ramus and Peletier, what matters in mathematics is the mental faculty corresponding to it. Mathematics has no special object or *genos* (since it is common to several disciplines), but a very special content, namely the intuitions which derive from human nature, which together with the counting faculty enable human beings to build mathematical sciences. Peletier and Ramus share the same view of mathematics, which interprets the particular value of mathematical demonstration in terms of their content and not of their form. For both, mathematics proceeds from a basic human faculty, called *mathesis* by Ramus. The particular strength of mathematical arguments and statements rests upon the status of *mathesis* with respect to truth, it being closer to the very basis of human knowledge.

The difference between the two groups of authors is also mirrored at the social level. Because of his unorthodox interpretation of Aristotle, Ramus had to struggle all his life. He was moreover a well-known Protestant who was eventually slain in the Massacre

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[56] About Schegk, Ramus and Kepler, see G. Cifoletti, "Kepler's 'De Quantitatibus'," *Annals of Science*, 43 (1986), 213-238, as well as André Robinet, *Aux sources de l'esprit cartésien* (Paris, 1996).

[57] Galen also contributed to the success of Stoic logic or dialectic in the sixteenth century. Galen could be used as an authority who followed Aristotelian ontology in logic, while also taking into account some crucial Stoic results. Notice that doctors needed mathematics, not least because of their reliance on astrology.

of St. Bartholmew's Day of 1572, the culmination of the French wars of religion. Similarly, Peletier was an intellectual dependent on patronage rather than on teaching, who had an impact mainly on what he considered the focus of his activity, namely printing. Dasypodius, in contrast, was a great mathematician, astronomer and clock-maker and as established in the Strasburg Gymnasium as Jakob Schegk was at the University of Tübingen.

Ramus' Scholae Mathematicae

Mathematics, in particular the philosophy of mathematics in sixteenth-century France, is traditionally connected to Ramus' name.[58] This is due to a large measure to his *Scholae mathematicae*. In book III of that work, Ramus develops a number of theses regarding Euclid's book II. As he sees it, the first ten propositions concern the problematic matter of the section of a segment, a section that is in fact one of the main ancient theories incorporated by Euclid into his *Elements*. It concerns mainly on the computation of segments (or application of areas), which Proclus would later consider common to both arithmetic and geometry. Ramus, following Proclus, maintains that the first ten propositions are indeed common to arithmetic and geometry, given that symmetry is proper to arithmetic, and through it arithmetic comes to geometry in 'species and reason'. And he adds that figure is proper to geometry, while figurate numbers are the interpreters of some of the features of the figures, though not of all, and that the arithmetic that deals with figurate numbers is not simply arithmetic, but arithmetical geometry (*arithmetica geometria*). It would appear that in this way, Ramus introduces into his reading of Euclid the premises of his 'geometric algebra' interpretation, which we mentioned before.

When he reaches proposition II, 4, Ramus translates thus from the Greek: "Si recta linea secta sit utlibet, totius quadratum aequale erit segmentorum quadratis et duplici segmentorum rectangulo," which may be rendered as follows: "If a straight line be cut at any point whatsoever, the square on the whole is equal to

58 On Ramus and mathematics, see the R. Goulding, "Method and Mathematics. Peter Ramus's Histories of the Sciences," *Journal of the History of Ideas* 67, (2006), 63-85.

the squares on the segments and twice the rectangle contained by the segments."[59] Ramus comments:

> This proposition compares a species with <its> genus and the species itself .Therefore it must come before 2 and 3.Theon also constructs the figures and then gives demonstrations of them, but is silent about the question of the equality posed. But this is also a corollary of I, 43, which is general with respect to this one. For, if a parallelogram is put equal to these four particular parallelograms, also the parallelograms around the diagonal of the whole square are squares, this is shown in prop. 24 of the VI book, which should precede this one, for it is general for parallelograms, hence it precedes it for us.[60]

This is a concrete example of a change in the order of Euclidean propositions. It is quite interesting that Peletier's version of the *Elements* is sensitive to the same 'logical' point concerning book I, proposition 43. In fact, Peletier's comment, of which passages have been quoted earlier, continues as follows:

> I call this figure 'mystical', because from it, as from an extremely rich promptuary, arise innumerable demonstrations. This is understood with great pleasure by those who seriously practice in the geometrical matter. For those who like the other's construction better, let the parallelogram BHFK be drawn first, then draw the parallel segment ED, then the parallel segment AG, and finally he can follow my proof.[61]

Peletier stresses here that this result will be from now on transmitted by the construction and used in several propositions and problems. Peletier here calls 'figure' something he has identified in a drawing which is worth considering not only with respect to arithmetic and algebra, but also for its own geometrical significance. For, in this case, what matters is not so much the graphic representation of a problem or a theorem, that is, the heuristic means of giving shape to the data, but rather the scheme that allows us to solve several problems and prove several theorems.

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[59] Ramus, *Scholae mathematicae*, 196. Notice that Peletier had translated proposition II, 4 differently: "Si recta linea secetur utcunque, Quadrata quae fiunt ex segmentis cum eo quod bis sub ipsis segmentis comprehenditur Rectangulo, aequalia sunt ei quod a tota fit Quadrato."

[60] Ibid.: "Haec propositio comparat speciem cum genere et ipsamet specie. Itaque praecedere debuit 2 & 3. Theon vero etiam figuras fabricatur, et eas demonstrat, tacet de positae aequalitatis quaestione. At hic etiam consectarium est e 43 p I, quae generalis ad istam fuerit. Parallelogrammum si quidem aequatur quatuor eiusmodi particularibus parallelogrammis, et diagonalia quadrati totius esse quadrata 24 p 6 demonstrat, quae hanc praecedere debuit, generalis quippe de parallelogrammo, ut nobis item praecedit."

[61] Peletier du Mans, *In Euclidis Elementa*, 41: «Hanc ego Figuram mysticam soleo vocare: Ex ea enim, velut ex locupletissimo promptuario, innumerabiles exeunt Demonstrationes.»

Should we call this, in modern terminology, a second degree figure? Insofar as it activates a chain of reasoning, it can be seen as the intermediate premise of a syllogism. Let us say that, like right angles, right triangles or similar triangles, parallelograms around a diameter and gnomous are a useful tool for finding solutions. In this case the figure goes beyond the visual description of a problem and grasps an aspect of its 'functioning', that is of the mathematical connections to be discovered between known mathematical entities. In fact, the use of this construction appears in connection with propositions 3, 4 and 5 of book II of the *Elements*, that is, those that Peletier, Ramus, and modern mathematicians alike, understood as solutions for the three cases of second degree equations. Whether inspired by Ramus or, conversely, inspiring the latter, Peletier's treatment of the gnomon left a legacy, given that even Clavius, who did not necessarily agree with him, included and developed it further in his edition of the *Elements*.

In the *Scholae mathematicae*, book III, Ramus evokes Proclus as the main resource in the project of reforming the faulty logic used in Euclid's *Elements*. He asks: "What does Proclus propose in order to construct the discipline properly?"[62] In fact Proclus, in his first *Prologue*, deals with some criteria for mathematical demonstration, which can be interpreted as his way of discussing Aristotle's three laws and that would later also define Ramus' method: *katà pantòs, katà autò* and *kath'olou protou*. They are the conditions for the axioms taken from Aristotle's *Posterior Analytics* as described in Ramus' *Dialectique*:

> L'axiome est du tout (vrai généralement)
> L'axiome est homogène.
> L'axiome est réciproque.[63]

Ramus writes in the *Scholae*: "The demonstrative method, says Proclus, is the passage from principles to the things being enquired."[64] For Ramus, as for Proclus before him, the method is the enthymeme, a demonstration which avoids the second premise.

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[62] Ramus, *Scholae Mathematicae*, 80.
[63] Ramus, *Dialectique*, 85.
[64] Ramus, *Scholae*, 81: «Demonstrativa methodus, inquit Proclus, est transitus a principiis ad quaesita.»

Common Notions in the De arte magna

Our discussion of the rhetorical reform of logic and its impact on sixteenth-century mathematics has led us from the use of Proclus' *Commentary* to the interpretations of book II of Euclid's *Elements*. The end of this process was constituted by Viète's discussion of axioms and *symbola* for the new analytic art. Having absorbed Ramus' lesson on Proclus, Viète decided to extend the list of Euclid's original axioms and to include the axioms on proportions related to them. In this manner, he founded a new art of mathematics capable of dealing with a general kind of quantity, number or magnitudes.

From the logical point of view, with the *symbola* he had the principles on which to build a global analytic art for all quantities. This is the origin of the logical foundation for the hypothetico-deductive construction described in the *Isagoge*.

But let us see first what Viète's protegé Guillaume Gosselin did with respect to *symbola* in his algebraic work of 1577. For his algebra, Guillaume Gosselin gives the three following axioms:

I. Things equal to the same thing are equal to each other.
II. If we add the same things to equal things, they will still be equal.
III. If we take equal things from equal things, the remainders will be equal.[65]

Gosselin offers us here exactly the same reduced list of axioms that Proclus had attributed to Hero. The first axiom allows him to give a proof of the solution formula of the equations of the form (in our terms):

$X^2 = b\ X$ or
$X^3 = b\ X$

…"and so on," as Gosselin writes. Notice that the canons, or cases for the first degree equations, were at the time:

$bx = x^2\ ;\ c = x\ ;\ c = x^2$

Gosselin calls this the 'simple equation' and considers it just an application of the first axioms. As for the 'composed equation', that is, the second degree equation, there are three cases or canons:

[65] Guillaume Gosselin, *De Arte magna, seu de Occulta parte numerorum que algebra et almucabala vulgo dicitur* (Paris, 1577), 54-55.

$x^2 + bx = c$; $bx + c = x^2$; $x^2 + c = bx$

Gosselin's proof rests again on Euclid. For instance, Gosselin gives the following arithmetical demonstration of the first canon:

10 L p 1 Q aequalis 56
that is
10 L + 1 Q = 56

let the value of L to be the number A
$10 A + A^2 = 56$

by *Elements* II, 1 :
$10 A = 5 A + 5 A$
$A^2 + 2 (5 A) = 56$

By *Elements* II, 4 :
$(5 + A)^2 = (5^2) + A^2 + 2 (5 A)$
thus:
$(5 + A)^2 = 56 + (5)^2$
$(5 + A)^2 = 81$
$5 + A = 9$
$A = 4$

In order to give the demonstration of a canon of the 'composed equation', as in the *zététique* 2 of the second book of the *Zététiques*, Viète is in the position of being able to count on his logical construction, his framework of symbola and of *Zététiques*.[66] In this case, the first *zététique* is based on an antithesis or the transposition of signs. We should therefore, also for Viète, give an entirely arithmetical demonstration. Notice, however, that Vaulezard precisely in this instance feels the need of adding, by way of an exception, two demonstrations, one with a gnomon, and the other described as "en lignes."[67]

Conclusion

The introduction of algebra into the mathematical sciences required many adjustments, during which the original art turned into a scientific discipline. One of them was the creation of a mythical origin for this non-classical procedure. Another was the development of a mode of presentation compatible with university curricula, whether in Latin or in the elevated vernacular. In this

[66] Ibidem, pp. 57-58.
[67] In his version of Viète's *Zététiques*; see J.L. Vaulezard, *La nouvelle Algèbre de M. Viète* (anastatic edition; Paris, 1986).

article, we have studied the problem of making algebra fit into the logical or theoretical structure of mathematics. We have seen some aspects of the striving for demonstration in algebra. Our authors did not content themselves with generic references to Euclidean geometry as contained in the tradition since Pacioli. They sought for stronger roots in Euclid's *Elements* themselves. This was done, first, by interpreting book II of Euclid's *Elements* as containing a doctrine on 'general quantity', which could then justify the algebraic formulas of resolution, and, second, by modifying the very structure of demonstration. We have seen that although Peletier and Ramus worked differently, they shared in this respect a common view of mathematics. In Peletier's commentary on book II, for instance, we see an interpretation close to Ramus' concerning the use of quadrilateral figure and syllogisms. As for Ramus, he deals in the *Dialectique* only with syllogisms because his definition of syllogism is Aristotle's definition of enthymeme.

As for the heirs of the French algebraic tradition, Gosselin and Viète, we mentioned that Gosselin was the first to introduce axioms for algebra in his *De arte magna*. This enabled him to give an *arithmetical* demonstration of algebraic solution procedures. In the process, he relied on Euclid's axioms; and this he could do because for him the interpretation of Euclid in terms of general quantity appeared self-evident. Gosselin's patron François Viète went even farther in the use of axioms. With his *symbola*, he rendered the basis of the scaffolding of proofs provided in his *Zététiques* explicit. But in his case, we do not need to argue very much for our thesis of the dependence of algebra on rhetorical logic, for he himself makes it quite explicit in the first lines of his first algebraic work, *In artem analyticen Isagoge*. He there calls algebra an 'analytic art' with reference to the analytical mathematical corpus. To be sure, the analytical way of doing mathematics, with its stress on problems, could easily be identified with algebra. But it was also seen as intrinsically connected to the rhetorical reform of logic, so that when Viète writes "what truly pertains to the *zetetic* art is established by the art of logic through syllogisms and enthymemes," he must be understood as addressing strictly logical matters. As far as mathematics was concerned, he had discovered the 'art of finding'; but, for logic, he had discovered the algebraic art of thinking and solving all problems.

LEIBNIZ ON MATHEMATICS, METHODOLOGY, AND THE GOOD: A RECONSIDERATION OF THE PLACE OF MATHEMATICS IN LEIBNIZ'S PHILOSOPHY

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Abstract

Scholars have long been interested in the relation between Leibniz, the metaphysician-theologian, and Leibniz, the logician-mathematician. In this collection, we consider the important roles that rhetoric and the "art of thinking" have played in the development of mathematical ideas. By placing Leibniz in this rhetorical tradition, the present essay shows the extent to which he was a rhetorical thinker, and thereby answers the question about the relation between his work as a logician-mathematician and his other work. It becomes clear that mathematics and logic are a part of his rhetorical methodology, because they constitute one set of tools that he used to excavate the truth. Mathematical and logical insights are thus all part of his "art of thinking," employed in the service of philosophy.

Introduction: A Leibnizian Puzzle

Scholars have long been interested in the relation between Leibniz, the metaphysician-theologian, and Leibniz, the logician-mathematician. For historians of science and for a long line of analytic philosophers starting with Bertrand Russell, it has been very tempting to think that the brilliant promoter of the universal characteristic and inventor of the calculus was *first* a logician-mathematician and *then* a metaphysician. In particular it is tempting to believe that the elaborate metaphysics of the *Monadologie* developed from Leibniz's ideas about truth, universal characteristic, and the continuum.

Given our concern with rhetoric and mathematics, the most relevant parts of the traditional story about Leibniz's intellectual development are as follows. (1) The only thing really original in Leibniz's very early years (1663-72) is his work in the areas of logic and the universal characteristic (besides some juvenile proposals in physics, which he would soon discard). (2) His main concern in Paris (1672-76) is with mathematical matters resul-

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ting in the invention of the calculus in 1675-76. (3) Soon after settling in Hanover (his primary residence from late 1676 until the end of his life), this original work in logic and mathematics encouraged the development of his theory of truth (as concept containment), whose textual evidence appears in 1679-83 and whose breakthrough encouraged major parts of his metaphysics. (4) The development of his notion of force (*vis viva*) in the late 1670s also encouraged a reconsideration of metaphysical matters and thus influenced the development of his metaphysics. (5) The *Discours de métaphysique* of 1686 is the first presentation of his metaphysics. (6) Throughout the 1690's and early years of the eighteenth century, he reconsidered the details of his metaphysical proposals, added to his physical insights, and wrote some theology for his royal patrons (the *Theodicie* of 1710), until he arrived at his "mature metaphysics," as most accurately presented in the *Monadologie* of 1714.

This traditional story is a satisfyingly tidy tale about a rational process from logic through mathematics and physics to theology and finally to mature metaphysics by one of the greatest philosophers, mathematicians, and scientists of the early modern period. Hard biographical facts support each part of the story, and in the past two decades, prominent scholars have endorsed those parts.[1] Concerning part (1), it is true that Leibniz does not argue, in any straightforward fashion, for a metaphysics in the very early years. Concerning (2), most of his energies in Paris are given over to mathematical matters, and the most important product of the period is the development of the calculus. Concerning (3) and (4), during the years 1679-83, he composed some highly original logical papers and made a significant breakthrough in the science of dynamics. Concerning (5), the *Discours* of 1686 is the first fully developed account of his metaphysical doctrines and their interrelations. Finally, concerning part (6) of the story,

[1] Among the prominent Leibniz scholars who have argued for some part of this story are: Robert M. Adams, *Leibniz: Determinist, Theist, Idealist* (Oxford, 1994); Philip Beeley, *Kontinuität und Mechanismus* (Stuttgart, 1996); Daniel Garber, "Leibniz: Physics and Philosophy," in Nicholas Jolley, ed., *The Cambridge Companion to Leibniz* (Cambridge, 1995), 270-352, and his yet unpublished Isaiah Berlin Lectures, Oxford, 2004; André Robinet, *Architectonique disjonctive, automates systémiques et idéalité transcendantale dans l'oeuvre de G.W. Leibniz* (Paris, 1986); R. C. Sleigh, Jr., *Leibniz and Arnauld: A Commentary on their Correspondence* (New Haven, 1990); and Catherine Wilson, *Leibniz's Metaphysics: A Historical and Comparative Study* (Princeton, 1989).

he did not publish any extensive work in theology until the *Theodicie*, dedicated to his royal patron, Princess Sophie Charlotte, in 1710, and moreover the *Monadologie*, which has become the canonical Leibnizian text, was indeed written at the very end of his life, as a summary of his philosophy.

Leibniz's intellectual path and scientific contributions however are neither as tidy nor as cumulative as this traditional story maintains. Indeed, once we begin to place both the path and the contributions within the context of "the art of thinking mathematically," a very different story emerges. In the introduction to our volume, Cifoletti offers a brief account of the "historiographical 'air du temps'" out of which the studies of the volume arise. The traditional story about his intellectual development reveals more about twentieth-century historiography than about Leibniz's intellectual evolution and thereby exemplifies the traditional "*querelle*" of "two cultures." For twentieth-century historians, the divide has been between Leibniz's "scientific" productions—the scientific achievements, the philosophical summaries, the texts full of definitions and arguments—and everything else. But once we consider the "other" culture and place the facts that support the traditional story alongside the enormous complications of Leibniz's life and texts, each part of the traditional story becomes either misleading or patently false.

One might list several hundred facts that do not easily mesh with the standard tidy tale, but here are a few, chosen to suggest the complications and diversity of Leibniz's intellectual production and to show the inaccuracy of each part of the traditional story.

(1) According to the standard story, the only thing really original in Leibniz's very early years (1663-72) is his work in the areas of logic and the universal characteristic. But once we consider the context of his proposals about the universal characteristic and about logical matters, it is obvious that they are a mere part of a more general metaphysical project. For example, his famous *Dissertatio de arte combinatoria* of 1666 begins with a number of metaphysical proposals that situate the text's logical claims. Not only does Leibniz present a Platonist account of divinity in the text, he offers an analysis of the four Aristotelian primary qualities in mechanical terms. In short, Leibniz places his original logical claims within a rather elaborate metaphysical context. This original glimpse at Leibniz's logical talents suggests that it

is inappropriate to treat his logical work in isolation from his other intellectual projects. Leibniz did not see his logical researches as distinct from the other parts of his thought (including his views about the divinity). It seems reasonable not to treat them as distinct.

(2) According to the traditional story, Leibniz's main concern in Paris (1672-76) is with mathematical matters resulting in the invention of the calculus in 1675-76. It is true that most of his energies in Paris were applied to mathematics. But he also found the time for his first full-fledged discussion of the problem of evil (in a dialogue entitled *Confessio philosophi*), an account of philosophical methodology, and a brilliant series of notes, entitled *De summa rerum*, on topics related to the metaphysics developed prior to his arrival in the French capital. In brief, Leibniz was actively engaged with various interrelated philosophical projects in Paris.

(3) According to the traditional story, between 1676 and 1683, Leibniz worked on a number of logical problems out of which developed his account of truth and which led him to develop his metaphysics. In fact, the traditional story is accurate about the logical contributions, but inaccurate about the relation between this part of Leibniz's "scientific production" and the other elements of his thought. In April 1679, Leibniz produced a series of papers that develop a theory of truth and treat a number of questions related to formal validity. Underlying these discussions is the idea that an affirmative categorical proposition is true just in case the concept of its predicate is contained in the concept of its subject. He takes true propositions to signify "nothing other than some connection between predicate and subject" in the sense that "the predicate is said to be in the subject, or contained in the subject."[2]

Beginning with Bertrand Russell and Louis Couturat in the first years of the twentieth century and continuing until very recently, it has been a commonplace among scholars that Leibniz's quirky

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[2] It is noteworthy that he presents this account of truth in a paper entitled *Elementa calculi*. See *G. W. Leibniz: Sämtliche Schriften und Briefe*, ed. Deutsche Akademie der Wissenschaften (Berlin, 1923-), Series VI, Volume iv, part A, p. 197. Hereafter I refer to this collection as A, and abbreviate my references as follows: large Roman numerals = series number, small Roman numerals = volume number, Arabic numerals = page number.

metaphysics developed more or less directly out of his account of truth.[3] As Russell neatly makes the point: "No candid reader ... can doubt that Leibniz's metaphysics was derived by him from the subject-predicate logic." [4] Russell used his impressive philosophical and scholarly skills to survey a wide range of Leibniz's texts. Given Russell's own philosophical interests, it should not come as a surprise to learn that he almost exclusively studied the "scientific" writings and ignored the other "culture." He thereby set a precedent in Leibniz studies.[5]

In my *Leibniz's Metaphysics*, I prove that Leibniz had an elaborate metaphysics by the time he left for Paris (that is, fourteen years before the *Discours*) and that the theory of truth developed from that metaphysics. While it is true that his account of substance as a logical subject with a complete concept (most famously articulated in *Discours* § 8) evolved subsequent to the theory of truth, it remains beyond doubt that he developed major parts of his metaphysics of substance several years before the account of truth. In brief, the logical work of 1676-83 could not have motivated the metaphysics.[6]

There are good reasons to see Leibniz's work in logic as part of a more general concern with the acquisition, teaching, and presentation of truths. Leibniz moved to Hanover in late 1676 to be court advisor to Duke Johann Friedrich. During the years 1679-83, he studied chemistry, and made detailed proposals about administrative matters, including the expansion of mining in the Harz mountains. Encouraged by the duke and inspired by the multi-volume *Encyclopedia* of Johann Heinrich Alsted, Leibniz

[3] For Louis Couturat's position, *La logique de Leibniz d'après des documents inédits* (Paris, 1901), and especially "Sur la métaphysique de Leibniz," *Revue de métaphysique et de morale*, 10 (1902), 1-25. For the most concise statement of Bertrand Russell's interpretation, see the preface to the second edition of his book, *A Critical Exposition of the Philosophy of Leibniz* (Northhampton, 1967). For recent books that approach Leibniz's thought and its development in this traditional fashion, see Nicholas Rescher, *On Leibniz* (Pittsburg, 2003), and Garrett Thomson, *On Leibniz* (Belmont, CA, 2001).

[4] Russell, *Leibniz*, v.

[5] This approach and main parts of the standard story continue to be endorsed among some historians. E.g., see Rescher, *On Leibniz*, and Thomson, *On Leibniz*. But it has become much more common for scholars to examine Leibniz's thought through a somewhat wider lens. For recent examples, see two fine introductions to Leibniz's thought: Nicholas Jolley *Leibniz* (London, 2005), and Massimo Mugnai, *Introduzione alla filosofia de Leibniz* (Turin, 2001).

[6] Christia Mercer, *Leibniz's Metaphysics: Its Origins and Development* (New York, 2001), see especially chapters 3, 4, 7, and 8.

planned a large encyclopedia project at the very time he was occupied with logical topics. In the midst of his logical papers, we find texts like *Praecognita ad Encyclopaediam sive scientiam universalem*, written in the winter of 1678-79 (A VI iv [A] 133-4). Here Leibniz discusses organizing all knowledge in encyclopedic form. There is no reason to consider his logical work in isolation from these other projects.

(4) Concerning the fourth part of the traditional story, it has become a commonplace among Leibniz scholars to highlight the development of his notion of force in 1679 and to assume that this major discovery must have influenced his account of substance. That Leibniz radically changed his thinking about the nature of force and motion in 1678 is clear. As a young man, he had embraced mechanical physics, according to which the features of bodies are to be explained in terms of the broadly geometrical properties of their parts—whether these are tiny indivisible atoms or infinitely divisible stuff—whose configurations shift and change through motion, and whose motion changes through collision. When he published his two-part *Hypothesis physica nova* in 1671, Leibniz's "abstract" account of motion is offered in terms of the Hobbesian notion of conatus, defined here as "an indivisible, nonextended part of motion" and as "the beginning and end of motion" (A VI ii 264-65). In 1671, Leibniz agreed with Descartes that "all power in bodies depend on speed"(A VI ii 228). By the time Leibniz met Spinoza in The Hague in the autumn of 1676, he had begun to question features of this mechanical account, and in particular, Descartes' law of the conservation of motion. In the winter of 1677-78, Leibniz took some observations made by the Dutch mathematician, Christiaan Huygens, about impact, and transformed them into a notion central to his thought. He decided that force or power of action (rather than speed) must be conserved in collision between bodies. By January 1678, he had hit upon the proper account of this force: mv^2 (mass times velocity squared). Given the importance of this insight, it is odd that Leibniz did not publish any part of his findings until 1686, and even then, in his *Brevis demonstratio erroris memorabilis Cartesii et aliorum circa legem naturae*, he merely criticized Descartes' conservation principle and hinted at his own account.[7] Over the next few

[7] *Brevis demonstratio* was published in the *Acta Eruditorum* in March 1686. See A VI iv [C] 2097.

years, he was to work out the details of his dynamics, especially in response to Newton's *Principia mathematica* (of 1687).

The account of force is a dramatic break with Leibniz's earlier physics. But it did not entail a break with his metaphysics. The theory of force may be easily added to the metaphysical foundations built during the early period.[8] Leibniz was clever enough to design his notion of substance to accommodate comfortably different accounts of motion and activity. Once he had hit upon the fundamental structure of his conception of substance as an active and unified self-sufficient thing (which he did in 1670-71), it would be sensible to conceive it as something that could ground slightly different accounts of activity. And once we situate Leibniz's insight within his fundamental assumptions, namely, that physics is a science or kind of knowledge "secondary" to metaphysics, it is not surprising that a breakthrough in physics would *not* require a dramatic shift in metaphysical doctrines. In an essay of 1679-81, Leibniz begins with a definition of physics as "scientia attributorum corporis" and then goes on to explain that even "distinct" knowledge of such an attribute must be reduced to "Metaphysics and Mathematics," since physics is a "subordinate" science (A VI iv [C] 1981-2). Despite its genuine significance, the development of his account of force did not require a break with his past metaphysical claims.

(5) According to the fifth part of the tradition story, the *Discours* of 1686 is the first presentation of Leibniz's metaphysics. In 1686, while overseeing the construction of mining machines he had designed for use in the Harz Mountains, Leibniz was caught in a severe snow storm and took the opportunity to write a summary of his metaphysics for "the great" Antoine Arnauld. While the snowstorm might have encouraged the first thorough-going

[8] Scholars have long noted the importance of Leibniz's development of the notion of force in the late 1670s, and most have taken this to be the first major step in the evolution of his metaphysics. See, e.g., Robinet, *Leibniz*, pp. 128-38; Adams, *Leibniz*, p. 236; and Garber, "Leibniz's Physics," passim. Subsequent to my discovery of Leibniz's original metaphysics, scholars have been inclined to see the break-through in physics as less the beginning of his metaphysical evolution as an important shift in emphasis. This is the position, e.g., taken by Daniel Garber in his Oxonian *Isaiah Berlin Lectures* in 2004. It has long been my contention, however, that if we take seriously the fundamentally active nature of Leibniz's original metaphysics, then it becomes easy to see that the account of force (in fact, virtually any mathematically describable account of activity) can easily be added to his original account of substance. See Mercer, *Leibniz's Metaphysics*, chapters 2, 4, and 7.

summary of Leibniz's views, it did not contribute significantly to its creation. Many of the doctrines contained in the *Discours* appear in the early metaphysics. Thus, the *Discours* does not mark the *creation* of a metaphysical system, but rather a new willingness to make the system public.

(6) According to the final part of the traditional story, the *Theodicie* and *Monadologie* are uniquely important as the most accurate summaries of Leibniz's mature views about theological and metaphysical matters. These works are important, because they were produced at the end of Leibniz's long life, but neither work by itself is radically new as a presentation of his views.

Leibniz's study in Paris of the problem of evil foresees many of the ideas of the *Theodicie* and the metaphysics of the *Monadologie* uses slightly new terminology but also bears a striking similarity to many earlier views. The *Theodicie* has a uniquely important status as the largest single philosophical text that Leibniz chose to publish during his lifetime, but it is neither his first nor absolute last word on the problem.[9] The *Monadologie* as a canonical text is even more problematic. Leibniz did not publish it, nor did he seem to think of it as the most accurate account of his philosophical ideas. Its canonical status has more to do with its use by eighteenth-century Leibnizians and the need of subsequent generations to create a stable Leibnizian philosophy than with any supreme importance that, as far as we know, Leibniz attached to it.[10] The assumption has been that because it appeared at the end of his life, the text must be the final statement of a long progress to philosophical clarity. In fact, there is much in the text that is better explained and more thoroughly explored in earlier writings. In brief, if we put aside our assumptions about the cumulative nature of Leibniz's development, there is little reason to attach quite so much importance to the *Monadologie*.

We have arrived at a Leibnizian puzzle. Once we cease to project our own assumptions about Leibniz's scientific "culture," there appears to be much more diversity to his intellectual life. That is, once we place his "scientific productions" within the

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[9] See R.C. Sleigh, Jr., *Confessio Philosophi: Papers Concerning the Problem of Evil, 1671-78* (The Yale Leibniz) (New Haven, 2006), Introduction.

[10] On the publication of the *Monadologie*, see Antonio Lamarra, Roberto Palaia, and Pietro Pimpinella, *Le Prime Traduzioni della Monadologie di Leibniz (1720-21)* (Florence, 2001).

elaborate complex of his theological concerns, political activities, and modes of intellectual exchanges, his story and the scientific contributions within it form a much less coherent and tidy tale. On the one hand, stands one of the greatest scientists of early modern Europe, someone who made contributions in logic, mathematics, physics, philosophy, and mechanical engineering. On the other hand, looms a somewhat enigmatic figure, someone who was court advisor, historian, city planner, librarian, engineer, expert in jurisprudence, and politician obsessed with religious peace, a thinker always prepared to present and contemplate his ideas from different perspectives. Where is the harmony in this diversity?

In the introduction to this volume, Cifoletti claims that once we "expand" our sense of rhetoric to match its use in the early modern period, it becomes "an important tool by which to dissect significant episodes in the history of early modern science." I would now like to use this tool as means to examine the relation between Leibniz's mathematics (generally conceived) and his methodology, and thereby to solve our Leibnizian puzzle. In the remainder of this paper, I take Cifoletti's advice to heart and consider Leibniz's thought within the larger methodological context of early modern rhetoric. Leibniz did not make explicit use of classical rhetorical devices, but he did practice what we are calling here "the art of thinking." Once we expand our sense of rhetoric to include the constant exploration of ideas and their relations, then the nature of Leibniz's life and its intellectual products look significantly different. As Cifoletti puts it in our introduction: as a tool, rhetoric "allows us both to enter more thoroughly the intellectual context of early modern agents and to situate ourselves at the level of the scientific work itself." The "scientific work" that interests me here is Leibniz's mathematical work in the Paris period, namely, the period of the development of the calculus. In order "to situate ourselves at the level" of that work, we need to do two things: first, to explicate Leibniz's rhetorical methodology, and second, to review the metaphysical doctrines that underlie his mathematical work.

Rhetoric in the Pursuit of Peace

In the aftermath of the Thirty Years War (1618-48), whose battles were fought mostly on German soil, a methodology of peace was extremely attractive, especially to German thinkers, many of whom had witnessed the devastation and horrors of the war first-hand. Born in Leipzig in 1646, Leibniz was raised in the aftermath of the war. He was educated in Leipzig and Jena whose professors were committed to the construction of a philosophical methodology that would generate intellectual harmony. For Leibniz's professors—philosophers like Jakob Thomasius, Johann Adam Scherzer, and Erhard Weigel—every intellectual tool that could be used in the pursuit of the intellectual harmony should be employed.

As a young man, Leibniz endorsed a rhetorical methodology and intended to develop a philosophy that would effect intellectual, religious, and even political peace. In this section, I display the rhetorical features of Leibniz's philosophy.[11] His rhetorical approach has (at least) four aspects worth noting: (1) from the beginning of his philosophical career, he sees himself as a synthesizer of ideas and traditions, and identifies with those philosophers who are similarly committed; (2) he is concerned with the consideration and comparison of various distinct intellectual sources as a means of arriving at his own philosophical insights; (3) he is prepared to formulate and reformulate these insights as a way of understanding them more thoroughly and connecting them to other ideas; and (4) he is keen to engage his readers and interlocutors so that they will join him in his conciliatory effort. I treat each of these features of Leibniz's thought in turn.

As a student in Leipzig, Leibniz acquired a conciliatory methodology and began to identify himself with those philosophers who attempted to find a unity among various philosophical doctrines.[12] He seems to have consumed ideas with a ferocious appetite and was happy to borrow from them whenever possible. The authors

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[11] Compared to other early modern philosophers who were wedded to more traditional views of rhetoric (e.g., those of Cicero), Leibniz seems to have forged his own approach. Until more research has been done on this part of his thought, we will have to be satisfied with these somewhat vaguely defined "aspects." I discuss features (1), (2), and (4) in *Leibniz's Metaphysics* (49-59). Thanks to the work of Cifoletti, however, I have acquired a broader understanding of these features.

[12] Particularly important here are Leibniz's professors in Leipzig, Jakob Thomasius and Johann Adam Scherzer. For more on their influence on the young Leibniz, see Mercer, *Leibniz's Metaphysics*, passim. For Leibniz's comments on the importance of conciliation, see, e.g., A VI ii 114, II i 176.

who interested him extend to ancient, medieval, Renaissance, and early modern figures, as well as to proponents of the new mechanical philosophy. The young man saved his most flattering remarks, however, for those intellectuals who promoted conciliation, and he criticized contemporaries like Descartes and Hobbes who did not.[13] As a synthesizer of ideas, the young Leibniz was eager to collect and compare ideas from a variety of sources. His works make it abundantly clear that the young Leibniz showed a keen interest in (what I have called) *conciliatory eclectics*, both past and present.[14]

The second aspect of his rhetorical approach is his concern to arrive at his own philosophical insight through the consideration and comparison of diverse intellectual sources. This is the source of his ideas and reveals one of the main roles of mathematics in his "art of thinking." As such, it deserves our full attention. Consider some comments that Leibniz made on a German scholastic text. In 1663/64 he took notes on Daniel Stahl's *Compendium metaphysicae*. Stahl (1585-1654) had been a well-respected professor in Jena, and his writings display philosophical acumen of a sort often not found in the textbooks of the period.[15] Leibniz's notes reflect his propensity to collect ideas (see A VI i 21-41). Although Stahl's book is a commentary on Aristotle's metaphysics, Leibniz brings an impressive range of authors and doctrines to the text. For example, he compares the views of Aquinas and Hobbes regarding Stahl's discussion of *ens* and *essentia* (A VI i 39). He thus compares ideas from a very wide range of sources.

Another example of the young man's early tendency to collect and combine ideas is the *Specimen quaestionum philosophicarum ex jure collectarum* of 1664. As the title suggests, Leibniz argues that students of jurisprudence cannot ignore metaphysics because in order to answer questions fundamental to law one must be acquainted with both divine and human matters. The young man gives a brief history lesson about jurisprudence, and then

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13 See, e.g., A VI ii 123, 433; II i 13.

14 In my *Leibniz's Metaphysics*, I say that, for Leibniz, "the true metaphysics will be constructed from the underlying truths in the great philosophical systems, will be consistent with Christian doctrine and the claims of the revelation, and will explain the phenomena (including the new experiments)" (53).

15 For a brief discussion of Stahl, see Max Wundt, *Die deutsche Schulmetaphysik des 17. Jahrhunderts* (Tübingen, 1939), 126-29. Stahl's *Compendium* is a very brief account of the components of the philosophy of Aristotle.

proceeds to discuss some of the great philosophical "mysteries" that are relevant to issues in jurisprudence. In his discussions, Leibniz collects ideas from the ancients (e.g., Protagoras, Plato, Galen), the late scholastics (e.g., Soto, Sanchez, Zabarella), Renaissance thinkers (e.g., Giovanni Pico della Mirandola), early modern conciliatory eclectics (e.g., Kircher, Alsted, and especially Grotius), and moderns (e.g., Hobbes, Gassendi). Leibniz constructs his answers from a wide range of sources and, through his methodological example, encourages the reader to seek a harmony beneath the intellectual discord and to build a firm metaphysical foundation for the study of jurisprudence. Nor is the *Specimen* Leibniz's only attempt during the period to speak to questions about education and learning. He proposes to reorganize the learning and teaching of jurisprudence. In his *Nova methodus discendae docendaeque jurisprudentiae* of 1667 he develops a philosophy of education that includes an analysis of the philosophical basis for law. In brief, he wants to reorganize the presentation of legal and ethical topics, make more perspicuous the underlying philosophical assumptions, and thereby promote knowledge and conciliation (e.g., A VI i 261-364).

Leibniz's early methodological assumptions are nicely exemplified in an edition of a text by the sixteenth-century humanist Mario Nizolio (1488-1567). He wrote a lengthy introduction to Nizolio's 1553 book, *De veris principiis, et vera ratione philosophandi contra pseudophilosophos*, in which he aligns himself with other conciliatory philosophers. Both Nizolio's text and Leibniz's introduction discuss the proper way of philosophizing. As part of his introduction, Leibniz attached a slightly revised version of a letter to his Leipzig professor and mentor Jakob Thomasius. The letter thereby became the young man's first published text on a contemporary metaphysical topic. In the letter, Leibniz insists that he is not just another philosopher "lusting for novelty," but desires to find the "interconnections among doctrines" (A VI ii 426). He intends to combine elements from diverse intellectual sources into a philosophy of conciliation, one that would effect peace and capture the "glory" of God's world. Nor did Leibniz waver from his conciliatory attempt to arrive at his own philosophical insights through a consideration of diverse philosophical traditions. In his *Nouveaux essays sur l'entendement humain*, written in the early years of the eighteenth century in response to John Locke's *Essay Concerning Human Understanding* of 1690, he offers a summary of his philosophy and the methodology that produced it. He writes:

> This system appears to unite Plato with Democritus, Aristotle with Descartes, the Scholastics with the moderns, theology and morality with reason. Apparently it takes the best from all systems and then advances further than anyone has yet done.... I now see what Plato had in mind when he talked about matter as an imperfect and transitory being; what Aristotle meant by his 'entelechy'; how far the sceptics were right in decrying the senses.... How to make sense of those who put life and perception into everything .... I see everything to be regular and rich beyond what anyone has previously conceived.... Well, sir, you will be surprised at all I have to tell you, especially when you grasp how much it elevates our knowledge of the greatness and perfection of God (A VI vi 71-73).

The third aspect of Leibniz's rhetorical approach to philosophy is his readiness to formulate and reformulate, try and retry his views as a means to unify and understand them more thoroughly. For much of the twentieth century, historians of early modern philosophy approached Leibniz as one of the great rationalist philosophers and sought to discover the ultimate truths of his system from which all the others could be deduced. They were not successful, and the result has been a reconsideration of the allegedly deductive structure of Leibniz's philosophy. Benson Mates was one of the first historians to make the point clearly. In his words:

> Contrary to what many commentators seem to have supposed, [Leibniz] does not treat his philosophical principles as a deductive system within which certain propositions are to be accepted without proof and the rest are to be deduced from these..... He deduces the various principles from one another in different orders and combinations. Often he gives alternative definitions of the same concept, sometimes even showing how to derive these from one another. It is obvious that he has no particular order of theorems and definitions in mind.[16]

In my *Leibniz's Metaphysics*, I make much of this feature of Leibniz's philosophical works in an attempt to highlight the difficulty of grasping the complications of his thought. Whereas our other early modern heroes—Descartes, Galileo, Spinoza, Hobbes, Malebranche—produced brilliant explications of their philosophies, there is no single exposition of Leibniz's metaphysics replete with extended arguments and details. What makes matters worse is that he often used different language and took diverse approaches

[16] Benson Mates, *The Philosophy of Leibniz: Metaphysics and Language* (Oxford, 1986), 4.

to the same topic. As Mates saw so clearly, Leibniz seems keen to place core doctrines in very different logical relations to one another.[17]

The most important aspect of Leibniz's rhetorical methodology given our concerns is his tendency to reformulate his philosophical opinions. Because he assumed that greater and greater knowledge would come through the persistent formulation and reformulation of his core doctrines, he was constantly eager to recombine the elements of his thought. Once he came to a position or had an insight, it did not remain a stable object. He seemed always inclined to review and reexamine. The most striking early example of this third aspect of his rhetorical approach is a series of notes he wrote between 1668-71, entitled *Elementa juris naturalis*. These notes are significant because they contain his original statements about universal harmony and his original use of the image of the mind as a mirror. What is especially important for us here is that they contain lists of definitions of key metaphysical, theological, and ethical notions, which are often arranged and rearranged. Similar definitions are reformulated in related texts. For example, there are various closely related proposals about how an individual created mind is related to its thoughts and to God, and subtly different accounts of justice and blame. We find various accounts of wisdom, love, and the pleasure associated with both. Often the same phrases appear in slightly different relations.[18] For example, Leibniz defines the mind and harmony in one manner and then slightly varies his account a few pages later (compare A VI i 438 and 444). Not only is Leibniz working out his views, he is playing with their interrelations.

Notes like these—which he made for most of his life—are large intellectual puzzles whose pieces can be reshuffled to produce different perspectives on the whole. Underlying the definitions in

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[17] However, neither Mates nor I are prepared to see this tendency as more than a difficulty that scholars must face. Leibniz's tendency to reinvent and reconsider ideas and their logical relations now seem a part his larger rhetorical strategy.

[18] At A VI ii 283, he writes: "Necesse est in cogitabilibus ipsis rationem esse cur sentiantur, id est cur existant, ea non est in singuloruom cogitatione, erit ergo in pluribus. Ergo omnibus. Ergo in Mente, id est uno in multis. Ergo in Harmonia id est unitate plurimorum, seu diversitate identitate compensata. Deus autem est unus omnia." Compare this formulation to others in the period, esp. those at A VI ii 487.

the *Elementa juris naturalis* we glimpse an account of knowledge according to which each part leads to knowledge of all the others. Leibniz explains: "What it is to have real knowledge, what is called in Latin *intelligere*, is to read the inner natures." For those who have "real knowledge," it is the good they ultimately seek. But what is this "real knowledge?" There is no "singular" knowledge without "universal knowledge." "The suggestion is that one must see the interconnections among things before any single thing can be known. It is not surprising therefore that he engages in a constant process of reviewing the logical relations and connections among things. Such engagement is the means to knowledge and ultimately to wisdom (and the good) (see A VI i 485).

The final feature of Leibniz's rhetorical methodology is that he intends to attract his readers and interlocutors to this conciliatory process of discovery. There are two assumptions here: the truth cannot be directly taught, and moreover, it will not be properly pursued unless the truth-seeker is engaged in the right way. Leibniz develops a strategy to entice the reader to consider the underlying (and often unstated) assumptions of his proposals. These he considers to be true and hopes that they will eventually lead his interlocutor to philosophical enlightenment and intellectual peace. In engaging his reader, Leibniz's first concern is to attain attention in the right sort of fashion.[19] In an unpublished note of 1669-70, he writes: "The power of persuasion consists sometimes in exhibiting reasons, sometimes in moving the affections; but at the heart of all these [means of persuasion] is of course the art of obtaining attention" (A VI ii 276). Along similar lines, he explains in 1668-69 that one of his theological demonstrations "has a three-fold use—to confirm those who think rightly, to attract the rest, and to prove philosophy to be a useful and necessary beginning for theology" (A VI i 514). The success of the true conciliatory philosophy to promote peace depends on its ability to *attract* students in a manner that will lead them to see the interconnections among the doctrines of their school and those of others. As Leibniz puts it to Hermann Conring, he extols the philosophical virtues of Aristotle to the Cartesians so

[19] Of all the aspects of Leibniz's rhetorical approach, this final one is classical. For example, see Cicero, *De inventione* 1, 23, on the importance of attaining attention in the right way.

as "to release them from the limitations of their teacher."[20] And as he explains in a letter to Duke Johann Friedrich of 1679:

> There are many sides to everything, and the way it [a philosophical proposal] is first seen determines much. The most harmless proposals have often been rejected on false suspicions, and the most scabby ones accepted through the ability of their supporters. People often do not take pains to examine matters thoroughly, and however acceptable views may be, they are sometimes rejected at once on a false prepossession (A II i 491).

During an unusually frank moment in 1676 he summarizes the point:

> A metaphysics should be written with accurate definitions and demonstrations, but nothing should be demonstrated in it apart from that which does not clash too much with received opinions. For in that way this metaphysics can be accepted; and once it has been approved then, if people examine it more deeply later, they themselves will draw the necessary consequences. Besides this, one can, as a separate undertaking, show these people later the way of reasoning about these things. In this metaphysics, it will be useful for there to be added here and there the authoritative utterances of great men, who have reasoned in a similar way; especially when these utterances contain something that seems to have some possible relevance to the illustration of a view" (A VI iii 573-74).

Cifoletti has shown that it was common for sixteenth-century thinkers to avoid putting their mathematical innovations front and center. Rather, they chose to introduce their new ideas by other means and to lead readers to insights by a slower method. Once we see Leibniz and his "scientific productions" in this tradition, it is not surprising that he often presents his ideas in such a tentative and conciliatory manner. Even this brief account of Leibniz's rhetorical tendencies offers us significant insight into the unity within the diversity of his intellectual pursuits. His concerns were as broad as the truth itself. His tentative conclusions, tendency to reconsider his ideas, and concern to capture "attention" in the right way are all features of his rhetorical approach. It is now time to turn our attention to the relation between his mathematics and this rhetorical process.

[20] G. W. Leibniz, *Die Philosophischen Schriften*, 7 vols., ed. C.I. Gerhardt (Berlin, 1875-90; repr. Hildesheim, 1978), I: 198. Hereafter G.

Rhetoric, Mathematical Knowledge, and God

The chronology of the relative development of Leibniz's major innovations in mathematics and metaphysics is now clear. Russell and his followers were mistaken in claiming that the mathematics and logic inspired the development of the metaphysics. If there is any influence from the one to the other, it must be the other way round. However, we need to be skeptical about the assumption that there is a neat progression even here. Assuming a relation between all the (apparently) disparate parts of Leibniz's thought, we might hypothesize that the mathematics and logic played a role in the on-going construction and then constant tinkering of his system. That is, the metaphysics encouraged his mathematical and logical works, and these "scientific" productions informed his metaphysics.

I would now like to look at some of Leibniz's mathematical and logical ideas in an attempt to situate accurately their place within the harmonized diversity of his thought. Leibniz turned to a variety of historical sources as a means of accessing the truth. Similarly, he approached mathematics for help in the construction of some of his most fundamental philosophical ideas. In the same way that he turned to traditional sources like the Aristotelian and Platonist philosophies for insights, he considered mathematics and logic as crucial tools in the discovery of the underlying truths in God's world.

When Leibniz arrived in Paris in 1672, he was relatively unfamiliar with recent mathematical findings and quickly set himself the task of catching up. It is all the more striking that he would soon make significant progress in the development of the differential calculus. At the very time that he was working so energetically on the calculus, he was making each part of the world infinitely complex. The simultaneous invention of the calculus and the infinite complexity of the world of creatures is an obvious point. The relations among the calculus, the related problem of the continuum, and the ontological "folds" of the world have been noted by others.[21] But our rhetorical tool al-

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[21] Although more work needs to be done, real progress has been made recently on this part of Leibniz's intellectual development. See Richard Arthur, *The Labyrinth of the Continuum: Writings on the Continuum Problem, 1672-86* (New Haven, 2001). Arthur's Introduction is particularly helpful on the relation between the attempt

lows us to discern something more profound underneath this obvious connection.

When we consider the rhetorical methodology underlying Leibniz's philosophical work in Paris, we glimpse its unity. His concern with theological matters (in particular his commitment that the world be a proper expression of God) encouraged his mathematical work, while his keen interest with infinitesimals and related matters informed his conception of divinely harmonized minds. It is not just that working on infinity in one place inclined Leibniz to apply it in another. Rather, as the rhetorical thinker he was, he thought it was a good thing—the methodologically right approach—to take a set of insights derived in one sphere and insert them directly into another. The point of the correct "art of thinking" is to find the connections among areas of study and extend those connections into new areas. To make the point another way, given his assumption about the unity of knowledge, it would have been perfectly reasonable to take an insight acquired in one area of knowledge and assume its relevance in another.

Once we realized that Leibniz was keenly interested in finding the unity among all the areas of knowledge and that he considered this goal the means to knowledge of God, the parts of his texts and his intellectual productions that have seemed disparate for so long suddenly seem much more unified. The parts are not, however, neatly arranged in a row. The world that God created is not, for Leibniz, *that way*. Rather, the world is a beautifully arranged, constantly changing expression of the divine attributes whose eternal natures may be glimpsed when the most focused rational minds make the relevant connections and find harmony among the diverse parts. Ultimately, it is the attributes that we hope to discover; and ultimately, it is the attributes that will lead us to God. But in the meantime, the attributes are best approa-

to solve the continuum problem and the metaphysics. Also see especially Samuel Levey, "Leibniz on Mathematics and the Actually Infinite Division of Matter," *The Philosophical Review*, 107 (1998), 49-96; Michel Fichant, *Gottfried Wilhelm Leibniz, La réforme de la dynamique: de corporum concursu (1679) et autres texts inédits* (Paris, 1994). Gilles Deleuze offers a playful though insightful account in his *Le Pli: Leibniz et le baroque* (Paris, 1988). Even less historically sensitive historians like Nicholas Rescher are aware that the "idea of creation as maximization, and the conception of an infinite comparison process along the lines afforded by the calculus." See Rescher, *On Leibniz*, 158.

ched by conceiving the interconnections and unities among the diverse parts of the created world.

As a conciliatory eclectic seeking a philosophy of peace, it makes sense to see Leibniz's mathematical and logical developments as contributing to that goal. And once we acknowledge that he embraced an emanative account of the relation between God and the world, according to which each creature in the world and the totality of creatures are manifestations of divine goodness and unity, it becomes easier to make out the exact place of mathematics in that system. Thus, before we turn to the connection between the rhetoric and the mathematics, we need to say a bit more about God.

God and Creatures, Harmony and Truth

Like other prominent thinkers of the seventeenth century, Leibniz believed in a perfectly good Supreme Being who created and maintained the world and whose existence could be proven. Like many of his contemporaries, Leibniz owed a number of his assumptions about God as creator of the world to an ancient (mostly Platonist) tradition. From prominent professors at the University of Leipzig, Leibniz acquired a solid education in Platonism. The Platonist Plotinus (204/5-270 AD) was the primary inspiration behind the version of this ancient philosophical "sect" taught in Leipzig in the mid-seventeenth century. These are the Platonist assumptions particularly relevant here:

God and Emanation: There is an ultimately good, perfectly self-sufficient, and thoroughly unified Supreme Being, on which everything else depends and which itself depends on nothing. God's mind contains a number of Ideas or attributes (say, the Idea of Justice), which are the perfect essences of things (these are roughly based on Plato's theory of Ideas) and which are used as models for created things. The Idea or attribute of God is emanated to a creature in such a way that neither God nor God's attribute is depleted in any way, while the creature acquires the attribute, though in an inferior manner. The emanative process is continual so that a creature instantiates a divine attribute if and only if God emanates the attribute to the creature. For many Platonists, a corollary of this causal theory of emanation is that every product of the Supreme Being contains all the attributes (and hence the essence) of God, though the product instantiates each of those attributes in a manner inferior to the way in which they exist in the Supreme Being. Justice as conceived by God is perfect; justice as instantiated by Socrates is not. Leibniz summarizes the position in § 14 of the *Discours*: "it is evident that created substances depend upon God, who preserves them and who even produces them continually by a kind of emanation."

Plenitude and Sympathy: The divine essence is emanated to each creature and to the whole of creation. The principle of plenitude develops from the idea that the more of the divine essence in the world—and hence of being and goodness—the better. Although the principle of plenitude suggests that there will be as much diverse being as possible (the more being, the better the world), this diversity of being must also be properly unified (the more unity, the better the world). One of the results of this unity among the parts of the world is a cosmic sympathy. Here the idea is that each part of the world is "in sympathy" with all the others. In other words, the principle of plenitude was supposed to imply that God fills creation with as much being as possible and unifies those diverse beings as much as possible. Such a diverse and unified world was supposed to engender wonder, delight, and awe in human observers. At the end of his life, in the *Monadologie*, Leibniz agrees with the ancient philosopher Hippocrates who claimed that all things are in sympathy with one another: everything "is affected by anything that happens in the universe, to such an extent that he who sees all can read in each thing what happens everywhere, and even what has happened or what will happen, by observing in the present what is remote in time as well as in space" (§ 61).

These ancient Platonist assumptions about emanation, plenitude, and sympathy inform much of Leibniz's thinking about the world. They inspire his theory of universal harmony, many of his views about mind, and his views about the mirroring (and expressing) of created substances. Leibniz agreed with many of his theist predecessors that the created world is an effect that bears the appropriate markings of its perfect, divine cause. But he also goes beyond the standard theist by employing mathematical means to add significantly to the unity and diversity of the world. More than any other major figure in the history of philosophy, he made use of mathematical notions to maximize the goodness of God's world. It is now time to see exactly how Leibniz's work and interests in mathematics influenced the development of his metaphysics by giving him new means to capture the grandeur and beauty of God's world. He takes his work on infinitesimals and related mathematical notions, and attempts to fill the world with as much good as possible. For Leibniz, more than for any previous philosopher, the world is best *because* it is maximally infinite. There are not just worlds within worlds *in infinitum*, but each of these worlds contains an infinity of minds or substances, each of which perceives all the others at each of the infinity of moments in its eternal existence. By such means, Leibniz added impressively to the harmonized perfection of God's world. The art of thinking mathematically helped him do so.

The Young Leibniz on God, Logic, and Knowledge

In the construction of his own conciliatory philosophy, Leibniz differed from many of his German predecessors and contemporaries in the way he took mathematical notions and wove them into the fabric of his metaphysical thinking. On the one hand, he assigned logic and mathematical reasoning an important role in his system. He was surely committed to the proper use of logic and "mathematical reasoning" as a means to describe precisely the structure and nature of truth. On the other hand, he warned against relying on the "mathematical disciplines" too much. It is now time to consider the interrelationship between Leibniz's mathematical and logical research and the evolution of his metaphysics. As a preface to his work in Paris, it will be helpful to make a couple of points about the early connections between logic and metaphysics.

Scholars have often treated Leibniz's account of logic and truth independently of his views about God and emanation, but the two parts of his philosophy are intimately related. The divine Ideas are the source of all truths, human minds contain these Ideas innately, and so the analysis of truth will involve these Ideas. Leibniz begins the *Dissertatio de arte combinatoria* with a demonstration of the existence of God, and begins the next section with: "To begin at the top, Metaphysics treats being and the affections of being"(VI i 169-70). In what follows, he characterizes mathematics and arithmetic as a means of analyzing such affections. The assumption that underlies this original publication on logic and the combinatorial art is that such logical tools ultimately rest on more fundamental metaphysical truths.

As he makes the point in the *Elementa juris naturalis*, that with the help of mathematical tools, we have become "conquerors of the world." But despite our power in manipulating the world, we have neither "real knowledge" nor its accompanying happiness (A VI i 485). Nor will we have real knowledge until the mind understands that the attributes or Ideas of God are contained in our minds as objects of knowledge and in the world as features of created things. Logic and mathematics are important tools to use to discover these Ideas and the truths that derive from them. This set of epistemological assumptions persists in Leibniz's thought until the very end. In the *Monadologie* he observes that our mind contains "knowledge of eternal and necessary truths . . . thus in thinking of ourselves we think of being" and "of the immaterial and of God himself" (§ 29-30). Logic is a tool to decipher and unlock some of those truths.

Method and Minds in Paris

Leibniz's Paris period (March 1672—October 1676) was enormously productive. In the fall of 1672 he met Christiaan Huygens, who immediately recognized the young man's talent and guided his mathematical studies. Leibniz devoted himself to study and by the fall of 1675 had begun to lay the foundations of his differential calculus. During the period of this intensive work on mathematics, Leibniz found little time for philosophy. It is therefore significant that one of the few philosophical essays of the period is a summary of his views about methodological matters and the role of mathematics in the pursuit of ultimate knowledge.

In this essay, *De vera methodo philosophiae et theologiae ac de natura corporis*, composed in the period 1673-1675, Leibniz describes in detail both his philosophical intentions and his methodological strategy. In the process he presents the secondary position he assigns to "the mathematical disciplines." He begins this essay as follows: "As I turned in my zeal for knowledge from the serious study of the sacred texts and of divine and human law to the mathematical disciplines, and was soon delighted by the thoroughly luminous teachings of the latter, I came near to being caught on siren cliffs. For some wonderful theorems were revealed to me." When one approaches such theorems "with a purified mind," one experiences a wonderful harmony. But despite the thrill of mathematical matters, Leibniz soon realized that it is essential to use the mathematical disciplines in the right way. For example, although the mechanical philosophy with its geometry has effected greater control of nature, it has "conveyed little" about the ultimate "nature of things" (A VI ii 329). Leibniz summarizes his concern:

> For I reflected as follows: Geometry clarifies configurations and motions; as a result we have discovered the geography of lands and the course of the stars, and machines have been made which overcome great burdens.... But the science that distinguishes the just man from the unjust and through which the secrets of the mind are explained and the path to happiness is paved is neglected. We have demonstrations about the circle, but only conjectures about the soul; the laws of motion are presented with mathematical rigor, but nobody applies a comparable diligence to research on the secrets of thinking (A VI iii 155).

The seeker of knowledge will be successful only if mathematical matters are treated with the right degree of care. Indeed, the wrong attitude toward mathematics will incline the mind to atheism: "I saw dangerous expressions slipping into men's souls; they are a sort of mathematical larva from which arises a false philosophy" (A VI iii 156-7).

In order to acquire genuine knowledge, philosophers must combine the search for theological insights with the care and precision of mathematics. For Leibniz, the "true method" in theology is to be constructed out of the insights of the scholastics along with the approach of the mathematicians—unlike the new philosophers who have too long ignored theological questions and who would allow "the whole of scholastic doctrine" to be "rejected," and unlike the scholastics whose "admirable reflections" are in need of clarification "by a mathematically schooled mind" (A VI iii 156). Leibniz wants to combine the theological insights and "marvelous subtlety" of scholastics like Aquinas and Gregory of Rimini with "mathematical rigor" (A VI iii 155). His goal is to effect a philosophical revolution by combining the best of all current strategies. As he sees it, neither the scholastic nor the mathematical approach is adequate, and the failure of each has encouraged confusion. Leibniz calls his philosophy a "religious" one and promises that it will lead wayward souls to the truth.

For the purposes of this paper, it is important that, according to Leibniz during his Paris years, the primary importance of mathematics is its contribution to the proper understanding of universal harmony and God. He summarizes the point:

> Mathematical studies will be used partly as an example of more rigorous judgment, partly for the knowledge of harmony and of the idea of beauty, experiments on nature will lead to admiration for the author of nature, who has expressed an image of the ideal world in the sensible one, so that all studies finally will lead to happiness" (A VI iii 157)

According to Leibniz, God has expressed the divine nature in the world with the result that the world is a harmonized, beautiful whole; mathematics is crucial as a means to acquire knowledge of that harmony. Once Leibniz describes the "true method," he is prepared to discuss the nature of substance and body. That is, in *De vera methodo philosophiae et theologiae ac de natura corporis*, after presenting the proper use of mathematical reasoning and tools, he is ready to present the core of his metaphysics on the basis of which he intends to effect theological and philosophical peace. He writes:

> There are certainly many and important things to be said ... about the principle of activity or what the scholastics called substantial form, from which a great light is thrown on Natural Theology and ... the mysteries of faith. The result is that not only souls but all substances can be said to exist in a place only through the operation of their active principle, that souls can be destroyed by no power of body; and that every power of acting [*omnem agendi vim*] exists from the highest mind whose will is the final reason for all things, the cause being universal harmony; that God as creator can unite the body to the soul, and that in fact, every finite soul is embodied, even the angels are not excepted, in which the true philosophy is in agreement with the teaching of the church fathers; finally, that the appearances differ from a substance (A VI iii 158).

In my book, *Leibniz's Metaphysics*, I explicate the details of the notion of substance here as well as the account of God and the relation between the creator and created. For our purposes, however, we need only to highlight three features of the active principles of nature (what he sometimes calls *mens*, sometimes *forma substantialis*). These features are: their dependence on God, their place in universal harmony, and their indestructibility. We will consider each of these briefly.

It follows from the theory of emanation that each created mind or soul depends entirely on God. Moreover, every product of the Supreme Being contains all the divine attributes (and hence the essence) of God, though the product instantiates each of those attributes in a manner inferior to the way in which they exist in the Supreme Being. From the beginning of his philosophical career, Leibniz associates activity with mind. Whether he calls these principles of activity minds, substantial forms, or monads, the idea is always that the only sources of activity in the world are divine-like principles that have the power to generate unity, self-sufficiency, and vitality. In a note of 1671, he argues: "Just as God thinks things ... because they follow from his nature, so does Mind.... Mind and God do not differ except that one is finite and the other infinite" (A VI ii 287-88). In the *Monadologie*, he notes "that souls, in general, are living mirrors or images of the universe of creatures, but that minds are also images of the divinity itself, or of the author of nature, capable of knowing the system of the universe…each mind being like a little divinity in its own realm" (§ 83). We also find there a neat summary of the relation between God and creatures:

> Thus God alone is the primitive unity or the first [*originaire*] simple substance; all created or derivative monads are products, and are generated, so to speak, by continual fulgurations of the divinity from moment to moment, limited by the receptivity of the creature, to which it is essential to be limited (§ 47).

Leibniz first articulates the doctrine of universal harmony in the *Elementa juris naturalis*. As he summarizes the idea for Arnauld in 1671: "I define … harmony as diversity compensated by identity" (A II i 173-74). By the time he wrote the *Discours* in 1686, he had come to formulate the doctrine in terms of hypotheses, though the underlying idea is still the same. In § 6 of the *Discours*, he explains: "God has chosen the most perfect world, that is, the one which is at the same time the simplest in hypotheses and the richest in phenomena." According to Leibniz, the single, unified, and perfect Supreme Being freely chooses to emanate the divine attributes to creatures; God remains transcendent while all creatures become an imperfect instantiation of God's attributes. Because God emanates the divine essence to all its products, Leibniz describes God as the reason (*ratio*) of the world and the one (*unum*) in it.

Universal harmony entails that God relates to the world and to each creature in it in two ways. God is the multiplicity in the world insofar as the divine essence is variously manifested in the vast diversity of creatures and in the diversity of the perceptions of each creature, but God is also the unity insofar as each created thing is a unified instantiation of the divine essence (although a manifestation of the essence far inferior to that of God) and therefore related to and reflective of all the others. The world is full of various perceptions of the world or 'phenomena', because the world contains an infinity of different expressions of the divine essence. Leibniz's notion of universal harmony forms the basis for his mature theory of preestablished harmony. At the center of that account stands a (harmonized) diversity of active and perceiving minds.

For a short period in 1670-71, Leibniz distinguished between the momentary minds in nature and the persistent minds of conscious, rational beings. His two-part published treatise, the *Hypothesis physica nova* of 1671, employs momentary minds as the cause of the motion in bodies to great effect. By 1676, he has decided to make all minds eternal: "every mind is of endless duration" and "is indissolubly implanted in matter.... There are innumerable minds everywhere" which "do not perish"(A VI iii 476-7). All minds act constantly and constitute self-sufficient beings that are eternal and indestructible by anything but God. Human minds are created by God and then exist eternally. Nonhuman minds exist from the beginning of the world to its end. Despite appearances to the contrary, the dog does not die, but shrinks down to "an invisible core" of substance from which it activates another substance, and so on for all of eternity. This remained Leibniz's view. As he writes in 1716: "there is never total generation nor, strictly speaking, perfect death, death consisting in the separation of the soul. And what we call *generations* are developments and growths, as what we call deaths are enfoldings and diminutions" (*Monadologie* § 73).

In brief, during the last several months of his stay in Paris, Leibniz is both engaged with his mathematical work and committed to the three features of minds just noted, namely, that minds are causally dependent on God, the source of harmony in the created world, and naturally indestructible.

Plentitude

Soon after making his breakthrough in the invention of the differential calculus, Leibniz applied his energies once again to metaphysical matters. It is here—in the period 1675-76—that we find the most significant examples of the interplay between his work in mathematics and his insights in metaphysics. During this time, each informed the other, resulting in a more unified and brilliant philosophical whole.

In the development of the differential calculus, Leibniz is keen to understand how to use the notion of infinity in the construction of mathematical entities such as the infinitesimals. The insight here encouraged him to reconsider the construction of metaphysical entities. From 1676 on, Leibniz is increasingly explicit about the significance of the principle of plenitude. In a series of notes written in Paris, entitled *De summa rerum*, he emphasizes its importance. For example, he writes: "I take as a principle … that the greatest amount of essence that can exist does exist" (A VI iii 472). He never wavers from this commitment to plenitude. In *De rerum originatione radicali* of 1697, he explains that God is the "reason" or source of things and argues that "there is a certain urge for existence or (so to speak) a straining toward existence in possible things or in the possibility of essence itself; in a word essence in and of itself strives for existence" (G VII 303).

For Leibniz, the world is not just very full, it is as full of being as it can possibly be, consistent with harmony. In 1676, Leibniz claims that every part of the world, regardless of how small, "contains an infinity of creatures" which is itself a kind of "world" (A VI iii 474: Pk 25). He emphasizes the same point later in *Primae veritates* of 1689-90: "*every particle of the universe contains a world of an infinity of creatures*" (A VI iv [B] 1647-48). For Leibniz, there is an aesthetic aspect to this elaborate harmony among the infinity of creatures. As he puts the point in the *Monadologie*:

> The author of nature has been able to practice this divine and infinitely marvelous art, because each portion of matter is not only divisible to infinity, as the ancients have recognized, but is also actually subdivided without end, each part divided into parts having some motion of their own; otherwise, it would be impossible for each portion of matter to express the whole universe" (§65).

Mirrors and Expressions

In *De summa rerum* of 1676, Leibniz develops his growing commitment to plenitude in a number of directions. By such means, Leibniz goes beyond the plenitude and sympathy of his Platonist predecessors. He does not just maximize creatures and the assumed sympathetic relations among them, he heightens their connections by making each substance a mirror of all the others, because each mind is (unconsciously) aware of all the others. [22]

[22] The image of the mind as a mirror is a permanent fixture of Leibniz's mature thought. He first develops this idea in the *Elementa juris naturalis* of 1670-71: "Since every mind is like a mirror, there will be one mirror in our mind, another in other

In *De summa rerum*, each mind eternally mirrors the entirety of the world, and each does so from its own perspective. For Leibniz, it is important that each mind has a unique view of the world for "in this way a wonderful variety arises" (A VI iii 524). As he summarizes the point in 1676: "A most perfect being is one that contains the most. Such a being is capable of ideas and thoughts, for this multiplies the varieties of things, like a mirror" (A VI iii 475). Forty years later, Leibniz sets out the same claims, employing the same analogies, in the *Monadologie*: "This interconnection or accommodation of all created things to each other, and each to all the others, brings it about that each simple substance has relations that express all the others, and consequently, that each simple substance is a perpetual, living mirror of the universe" (§ 56). As such quotations suggest, there are close connections between the mirroring activity of minds and Leibniz's mature doctrine of expression. In various texts and in various ways, Leibniz claims that each substance expresses God, each substance expresses the world, and each substance expresses every other substance. *De summa rerum* reveals the underlying motivation behind the doctrine. Each substance is an emanation of God's essence and in this sense each shares the same essence. Each emanation will differ from every other by *expressing* the divine essence differently: "The essence of all things is the same," and they differ "only in the manner of their expression"(A VI iii 573). Each substance—whether a human or snake—is a more or less clear expression of the divine essence. Leibniz concludes: "so do things differ from each other and from God" (A VI iii 519).[23] For Leibniz in 1676, each substance expresses God insofar as it expresses the divine essence; each expresses the world insofar as the world just is the totality of expressions of God; and finally, each substance expresses every other insofar as each is a more or less clear expression of the same thing. The *Discours* of 1686 also employs this notion of expression to great effect: "Every substance is like a complete world and like a mirror of God or

minds. Thus, if there are many mirrors, that is, many minds recognizing our goods, there will be a greater light, the mirrors blending the light not only in the [individual] eye but also among each other. The gathered splendor produces glory" (A VI i 464).

[23] To explain his point in the Paris years, Leibniz often uses an arithmetical analogy. See e.g. A VI iii 512, 519.

of the whole universe, which each one expresses in its own way, somewhat as the same city is variously represented depending upon the different positions from which it is viewed" (§ 9). He goes on to add that substances are "different expressions of the same universal cause, namely, God," where "the expressions vary in perfection" (§ 15).

Marks and Traces

The eternity of all mind-like active things is not an obviously plausible theory. Leibniz endorsed it because the eternity of minds adds significantly to the plenitude and harmony of the world. While developing his opinions about plenitude in *De summa rerum*, Leibniz hit upon the idea that each mind-like creature eternally perceives the entirety of the world. Each mind "senses all the endeavors" of all the other minds in the whole history of the world: "no endeavor in the universe is lost; they are stored up in the mind, not destroyed"(A VI iii 393). He came to believe that plenitude requires that each moment in the eternity of the world contain its whole history, past, present, and future. Minds not only sense all the present activities of all the minds in the world, they also retain a memory or "trace" of them: "it is not credible that the effect of all perceptions should vanish" (A VI iii 510). Each mind "retains the effect of what precedes it" and also "has a quality of such a kind as to bring this [state or effect] about" (A VI iii 491).

Thus, in 1676, Leibniz develops a version of his doctrine of marks and traces according to which each mind at every moment includes an effect or trace of all it has done as well as a quality or mark of all it will do. In § 8 of the *Discours*, he offers the soul of Alexander as an example: "there are vestiges of everything that has happened to him and marks of everything that will happen to him and even traces of everything that happens in the universe, even though God alone would recognize them all." By making minds eternal, allowing them to sense all endeavors, and assigning them traces of all that has gone before and marks of all that will occur, Leibniz makes each mind a mirror of the entire course of the world at every moment in time. That is, each mind reflects or mirrors the entire world at every moment of the mind's eternal existence. In *Discours* §15, he summarizes the point in terms of expression: each substance is of "infinite extension insofar as it expresses everything" (A VI iv [B] 1646). By such means, he agrees with Plato "who taught that our soul expresses God, the universe, and all essences"(§27).

Conclusion

Once we put aside our assumptions about Leibniz's scientific "culture" and place his scientific contributions within the wider context of his life and works, we begin to glimpse the harmony at the center of his intellectual life. That is, once we place his "scientific productions" within the elaborate complex of his theological concerns, political activities, and modes of intellectual exchanges, the story of his philosophical development becomes more elaborate than the traditional account would have us believe.

First and foremost, Leibniz was a rhetorical thinker: he intends to access the truths through the analysis and consideration of a wide variety of philosophical doctrines and then to persist in the reformation and reconsideration of the insights so attained. He does not intend to offer his insights too aggressively. Rather, he hopes to attract and then engage the seeker of truth in his "art of thinking." Mathematics and logic are a part of his rhetorical methodology because they constitute one set of tools that he uses to excavate the truth. Mathematical and logical insights are thus a part of his "art of thinking."

Leibniz's metaphysics did not develop out of his interest in mathematics and logic; nor did the mathematics and logic develop out of his metaphysics. Rather, his work in mathematics and logic developed hand-in-hand with his metaphysics. The core doctrines of the metaphysics are in place before he begins serious study of mathematics, and metaphysics was always more fundamental to his thinking than mathematics. But it remains true that each *informs* the other. In keeping with his conciliatory methodology, he took some of the mathematical ideas that most interested him – about the infinite and the construction of mathematical entities like infinitesimals – and applied them to his Platonist views about universal harmony, plenitude, and minds. In other words, the pursuit of mathematical insights is just another means to access the unity within the divinely arranged truths. In the same way that he uses other philosophical sources (e.g., the Platonist notions of emanation and sympathy), he uses mathematics as a means to knit together God's truths. In the same way that the Aristotelian and Platonist philosophies offer insights, mathematics and logic are crucial tools in the discovery of the underlying truths in God's world. Mathematics in general and the study of infinity and infinitesimals in particular give Leibniz a new way to capture the grandeur and beauty of God's world. The result is a brilliant melding of elements to form "the best of all possible worlds."

In some notes written during his stay in Venice in 1690, Leibniz summarizes his underlying assumption about the unity of knowledge:

> Each thing is so connected to the whole universe, and one mode of each thing contains such order and consideration with respect to the individual modes of other things, that in any given thing, indeed in each and every mode of any given thing, God clearly and distinctly sees the universe as implied and inscribed.

Due to this connection among things, "the more perfectly I perceive one thing, the better I come to know many properties of other things from it. And from this perfect consonance of things there also arises the greatest Harmony and beauty of the Universe, which exhibits to us the power and wisdom of the highest maker" (A VI iv [B]1668).[24]

[24] I would like to thank Giovanna Cifoletti for teaching me so much about rhetoric. Also thanks to Eric Brian for his kind support and good humor.

COMBINAISONS ET DISPOSITION LANGUE UNIVERSELLE ET GÉOMÉTRIE DE SITUATION CHEZ CONDORCET (1793-1794)

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Abstract

The recent edition of a long manuscript which Condorcet wrote in 1793 or 1794 in the form of an "Essai on a Universal Language" (*Essai d'une langue universelle*) provides a good occasion for analyzing certain implicit aspects of eighteenth-century mathematics. For Condorcet understood this 'universal language' as a technical method of universal application for all sciences and in all vernacular languages. Although this project was destined to remain without direct heirs, it is embedded in the mathematical skills of the age and linked to the tradition of a Hérigore, Leibniz or Turgot and to the attempt of early modern algebraists to develop a mathematical 'art of reasoning'. Those pages in Condorcet's manuscript that treat of the *analysis situs* document the importance attached to the art of mapping elements of thought, of combinatorics, and of the reduction of geometrical propositions to a sequence of signs. A telling example of this procedure is the distinction between a mere crossing of threads and a knot.

Le succès de posthume de l'*Esquisse d'un tableau historique de l'Esprit humain* (1795) de Condorcet (1743-1794), sa forme narrative allusive orientée vers un ouvrage bien plus ample – le *Tableau historique* lui-même –, sa teneur apparemment plus philosophique que mathématique ont suffi à écarter pour longtemps les historiens des mathématiques des papiers laissés par le savant. Au terme d'un renouveau des recherches sur Condorcet et sur son temps amorcé depuis le milieu des années 1970 et après une dizaine d'années de travaux menées par un groupe de chercheurs spécialisés[1], un vaste corpus – presque le quintuple du volume

[1] On suit l'itinéraire de ces travaux dans *Sciences à l'époque de la Révolution française. Recherches historiques*, sous la dir. de Roshdi Rashed (Paris, 1988); *Condorcet, mathématicien, économiste, philosophe, homme politique*, sous la dir. de Pierre Crépel et Christian Gilain (Paris, 1989); *Condorcet 1794-1994. Mélanges de l'Ecole Française de Rome. Italie et Méditérannée*, tome 108, 2 (Rome, 1996) ; *Condorcet, homme des Lumières et de la Révolution*, sous la dir. de Anne-Marie Chouillet et Pierre Crépel (Fontenay, 1997).

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initial de l'*Esquisse* – est paru en 2004 qui restitue aussi complètement possible ce qui reste de l'ouvrage résumé par son propre auteur : le *Tableau des progrès de l'esprit humain*. Il ne s'agira pas ici de présenter l'édition récente[2], ni de restituer l'atelier de Condorcet[3], ni même d'analyser la réception de l'*Esquisse* et des fragments du *Tableau* parfois parus dès le XIX^e^ siècle[4].

Dès 1795, les différentes éditions de l'*Esquisse* ont porté un lot considérable de malentendus philosophiques, scientifiques et politiques – plus précisément de raccourcis. L'*Esquisse* était un prospectus, c'est-à-dire une occurrence d'un genre littéraire ancien qui consistait à annoncer un ouvrage et à le faire valoir. Cette manière abrégée et programmatique a donné prise à une lecture téléologique simpliste et bien connue qui n'était pas présente comme telle dans l'entreprise du *Tableau historique*, pour autant que la récente édition permette d'en juger. On sait qu'aussitôt publié, le prospectus a suscité autant d'enthousiasme que de rejet ou de perplexité : sous cette fausse apparence, on le sait, il a révolutionné ce que Voltaire dès 1765 avait déjà qualifié de « philosophie de l'histoire ».

Le fragment sur la langue universelle

La Xe Époque du *Tableau* était réservée aux temps futurs. Parmi les fragments que Condorcet a préparé pour la composer l'un d'entre eux est demeuré inédit pour près des deux tiers : le *Fragment 4*[5],

[2] Condorcet, *Tableau historique des progrès de l'esprit humain. Projets, Esquisse, Fragments et Notes (1772-1794)*, publié sous la direction de Jean-Pierre Schandeler et Pierre Crépel par le groupe Condorcet (Éric Brian, Annie Chassagne, Anne-Marie Chouillet, Pierre Crépel, Charles Coutel, Michèle Crampe-Casnabet, Yvon Garlan, Christian Gilain, Nicolas Rieucau, Jean-Pierre Schandeler) (Paris, 2004), XVI-1332.

[3] Sur les conditions de rédaction de l'ensemble depuis les années 1770 et jusqu'à la période où le secrétaire perpétuel de l'Académie royale des sciences était proscrit, voir le *Tableau historique*, 1-174.

[4] Sur la réception de l'*Esquisse*, soir Jean-Pierre Schandeler, *Les Interprétations de Condorcet. Symboles et concepts (1794-1894)* (Oxford, 2000). Les recherches importantes menées sur la philosophie politique de Condorcet, depuis Keith M. Baker, *Condorcet. From Natural Philosophy to Social Mathematics* (Chicago, 1975), et jusqu'à David Williams, *Condorcet and Modernity* (Cambridge, 2004), peinent à saisir les liens entre le travail du mathématicien et son action politique ; en ce qui concerne ce lien pendant les dernières décennies de l'Ancien régime, voir Éric Brian, *La Mesure d'Etat. Administrateurs et géomètres au XVIIIe siècle* (Paris, 1994).

[5] Cette dénomination est de la plume de Condorcet.

dont le titre commence par « *Essai d'une langue universelle* »[6]. Condorcet a annoncé ce développement dans le *Prospectus* et dans ses manuscrits[7]. Il a même précisé son utilité, à ses yeux la plus générale[8] : l'humanité détruite en totalité ou presque, mais son œuvre préservée en lieu sûr et écrite dans une langue intelligible sans aucune communication antérieure, un être – humain peut-être, et en tous cas doué de raison – pourrait, tel un Platon imaginaire devant des inscriptions égyptiennes, déchiffrer le recueil de la science des hommes rédigé en langue universelle[9]. Le *Fragment 4* est ainsi tout à la fois le manuel et l'exemple d'une langue qui reste à élaborer.

> Le moyen le plus simple de faire bien connaître la nature d'une langue universelle et philosophique, et de montrer toute l'étendue des avantages qu'elle promet, serait sans doute d'en présenter quelques essais, d'en faire voir par l'exécution même la possibilité, les difficultés et l'usage (750r/957).

Son titre initial annonce une première partie dévolue aux sciences mathématiques et physiques, c'est à dire, comme à l'Académie royale des sciences à Paris dont Condorcet fut le Secrétaire perpétuel, à l'algèbre, à la géométrie, à l'astronomie, à la mécanique, à la chimie et à l'histoire naturelle. Plusieurs fois, Condorcet indique qu'il sera aussi question des sciences morales et politiques, de la métaphysique et de la logique mais ces domaines, tels qu'il les entend, ne sont considérées que rapidement (793r-v/1013-1014). Quoiqu'il en soit, il s'agissait bien pour le géomètre d'une langue dont les principes auraient valu indifféremment pour exprimer les unes ou les autres de ces sciences.

Si Condorcet a mesuré que son développement s'était en fait écarté d'un strict exposé des progrès possibles de l'esprit hu-

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[6] Bibliothèque de l'Institut de France, ms 855 (III) *folios* 750 *recto* à 795 *verso*. Nous en avons établi le texte pour l'édition de 2004. Il en occupe les p. 957-1014. L'édition annotée est accompagnée d'une introduction, p. 947-956, et d'un tableau récapitulatif des notations, p. 1015-1029. Dans la suite, les indications de foliotage et de pagination seront abréviées de cette manière : par « 774v/995 », il faudra entende ms 855 (III) *folio* 774 *verso*, page 995 de l'édition de 2004.

[7] *Tableau historique*, 455 et 918-919.

[8] *Tableau historique*, 995-996.

[9] 773r-775v/994-995. Condorcet songe à Platon – au *Timée*, 22c-23c notamment – mais c'est Solon, aux dires de Critias l'Ancien et selon Platon, qui se serait trouvé confronté aux réflexions d'un prêtre égyptien sur l'écriture, sur la mémoire des arts et des lois, et sur les périls des destructions auxquels l'humanité est soumise.

main[10], il a voulu faire en sorte que le fragment fournisse à ses lecteurs non pas un dictionnaire, ni une exposition de cette langue, mais le moyen de la former et de la développer. Il est donc un peu dérisoire de la décrire ne s'attachant qu'à son exposé inachevé : il faut au contraire la reconstituer, puis mettre cette reconstitution, d'une part à l'épreuve de ce que pouvait connaître Condorcet en son temps et, d'autre part à celle de sciences que nous connaissons. L'édition de 2004 a visé une restitution complète du manuscrit que les papiers laissés par Condorcet permettent de considérer comme pertinent à ses yeux au moment où il n'y reviendra plus.

Dans cet article, il s'agira d'offrir un élément de lisibilité au texte ainsi reconstitué. A vrai dire, sous l'angle qui nous intéresse en fait ici même, il ne sera pas question de tout le *Fragment 4* sur la langue universelle mais de quatre feuillets seulement – inédits jusqu'en 2004. Ces pages traitent de la géométrie de situation à la Leibniz. Notre étude s'inscrit dans le mouvement récent de l'historiographie des mathématiques qui consiste non pas seulement à établir *ce que l'on savait* à telle ou telle époque mais encore à saisir *comment on travaillait* et comment on raisonnait en mathématiques à cette époque. Le *Fragment 4* à cet égard est un matériau exceptionnel : c'est l'un des très rares documents où un mathématicien ancien de haut niveau a tenté de rendre tout à fait explicite ce qu'il faisait. Toutefois on mesurera à la lecture de cette étude qu'on est encore loin de pouvoir produire une analyse systématique du fragment, des généalogies intellectuelles dans lesquelles il s'inscrit et des perspectives qu'il procure sur les mathématiques de la fin du XVIII^e siècle ou sur les avatars ultérieurs dont les traces peuvent apparaître au fil du texte – autant de questions qui sont l'objet d'un ouvrage en préparation. Pour mieux comprendre ce document, il importe de subordonner la reconnaissance éventuelle de choses mathématiques connues aux XIX^e ou XX^e siècles à une meilleure restitution des arts de raisonner et des conceptions des rapports entre les langues et les mathématiques en vigueur du XVI^e au XVIII^e siècle[11].

L'ensemble du manuscrit autographe du *Fragment 4* couvre près de quatre-vingt-dix pages. Si parfois Condorcet a écrit au fil de la

10 *Tableau historique*, 468-469.
11 Voir l'introduction de Giovanna Cifoletti au présent numéro.

plume, le plus souvent il a terriblement repris son texte. Il peut y être revenu trois, quatre ou cinq fois. Les pages sont le plus souvent très peu lisibles, et la matière est passablement abstraite. Il ne faut donc pas s'étonner que le fragment n'ait pas connu jusqu'à ce jour d'édition complète. Deux tentatives ont précédé la publication de 2004. Au temps de la préparation des *Œuvres* parue au milieu du XIXe siècle[12], Eliza Condorcet-O'Connor, la fille de notre géomètre, a fait procéder à une copie, presque un fac-similé, qu'elle a relue attentivement. Mais elle n'a pas été satisfaite du résultat, il est vrai souvent fautif. Une note de sa main au crayon (699v) révèle qu'elle a soumis cette copie imparfaite au maître d'ouvrage de l'édition, le mathématicien et Secrétaire de l'Académie des sciences, François Arago. Probablement a-t-on hésité avant de renoncer à la publication : Eliza a tenté elle-même d'achever la copie. Le fait est que le manuscrit sur la langue universelle n'a pas été publié. En 1954, Gilles-Gaston Granger a publié dans la *Revue d'histoire des sciences* une édition des folios 750r à 761v, 786 v à 791v et un passage du 793v, soit les quatre dixièmes du fragment, et principalement la partie qui concerne la langue de l'algèbre[13]. Granger a contrôlé les deux copies préparées au milieu du XIXe siècle en les comparant à l'autographe. Il a ainsi corrigé nombre d'erreurs.

Entre 1794 et 1849, le *Fragment 4* n'eût probablement que trois lecteurs : Eliza, la personne qui a travaillé à la copie, et peut-être Arago. Il n'est toutefois pas impossible que des savants du début du XIXe siècle, tel Fourier, aient pu consulter les papiers laissés par Condorcet, mais rien ne permet de conclure qu'ils aient eu l'envie et le courage de déchiffrer l'autographe. Pendant le siècle qui a suivi, tout nous porte à croire que seuls l'historien Léon Cahen, au début du XXe siècle, et le philosophe Gilles-Gaston Granger, un demi-siècle plus tard, ont porté leur attention au dossier formé par l'autographe et les deux copies partielles. Depuis le milieu des années 1980, et en vue de l'édition du *Tableau historique* de 2004, une demi-douzaine de chercheurs y sont revenus. C'est dire que la langue universelle de Condorcet, comme le constatait déjà Granger, n'a eu aucune fortune ni dans l'histoire des mathématiques ni dans l'histoire de la logique et

[12] *Œuvres de Condorcet* (Paris, 1847-1849), 12 vols.

[13] Gilles-Gaston Granger, « Langue universelle et formalisation des sciences. Un fragment inédit de Condorcet », *Revue d'histoire des sciences*, 7 (1954), 197-219.

de la philosophie des mathématiques : personne à notre connaissance n'a travaillé dans la continuité directe de cette tentative, ni personne n'a indiqué explicitement ou manifesté clairement, dans des travaux postérieurs à 1954, sa connaissance de la tentative de Condorcet[14].

La fabrique de la langue

Comment rendre compte du projet de Condorcet[15] ? Granger a mis en évidence qu'il importait de ne pas abuser d'une lecture strictement formelle de ce manuscrit. Il ajoutait : « c'est dans le cadre d'une interprétation sociale de la connaissance qu'il convient tout d'abord de placer le texte inédit qui va être présenté » (p. 198). Effectivement, la publication complète du *Tableau historique* et du *Fragment 4* ont confirmé ce premier constat. Une théorie condorcétienne de la langue – plus précisément dans des termes actuels une épistémologie sociale et historique du langage scientifique – se consolide en effet tout au long des étapes de la formation historique du *Tableau* depuis les années 1770 jusqu'aux manuscrits des années 1790. Roselyne Rey a toutefois appelé l'attention des commentateurs sur le caractère inachevé du fragment et sur les difficultés qu'avait rencontré Condorcet à soumettre aux mêmes procédés des matières aussi diverses que l'algèbre, la chimie, la mécanique, l'astronomie et les sciences morales. Elle a souligné de plus le fait que la langue tracée au fil manuscrit est caractérisée par une orientation vers l'autoperfectionnement.

Il est vrai qu'en livrant avec le *Tableau historique* une synthèse de l'expérience scientifique humaine, Condorcet n'entendait pas simplement affirmer sa propre conception de la langue des sciences, mais livrer un ouvrage exemplaire de ce qu'il tenait pour *le propre de l'esprit humain*. Le *Tableau* comme la langue universelle procèdent à ses yeux non d'un souci historique et technique, mais d'une anthropologie de la connaissance où la langue, ses traces,

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[14] Parmi ceux qui ont travaillé sur ces manuscrits en vue de l'édition de 2004, il nous faut mentionner le nom de Roselyne Rey, disparue prématurément en janvier 1995. Nous avons publié ses premiers résultats dans le volume *Condorcet. Homme des Lumières et de la Révolution*, cité plus haut.

[15] Ce n'est pas le lieu ici de justifier l'établissement du texte. Les passages que nous citons s'appuient sur notre édition de 2004.

ses signes et ses supports tiennent une place primordiale. Le *Fragment 4* à cet égard fut comme une mise à l'épreuve de cette vision générale d'une langue universelle des sciences. Ses difficultés et ses incohérences sont l'heureuse démonstration qu'en matière de science et de langage, aucune équation – même posée en principe – n'épuise la question. Ainsi, dès qu'il traite de la partie de la langue universelle propre à une nouvelle science, Condorcet déploie de nouveaux aspects d'un art de réduire en écriture qu'il perfectionne au fil de la plume. De là un texte dynamique où la langue à construire évolue des premiers feuillets jusqu'aux dernières retouches. L'analyse fine de cette évolution doit passer par un examen des multiples variantes que les différentes couches du manuscrit laissent reconstruire. Des paragraphes, des pages entières sont biffées, puis réécrites. Leur matière est mûrie tout au long d'une centaine de pages. Condorcet tente parfois une récapitulation, preuve qu'il découvre lui-même, chemin faisant, les métamorphoses de la langue qu'il produit.

Bien qu'on dispose d'un nombre important de manuscrits de la main de Condorcet et d'une grande quantité d'imprimés, aucun d'entre eux ne paraît procurer les traces d'une entreprise antérieure comparable chez cet auteur, ni même une discussion explicite et nourrie d'anciens modèles, si ce n'est peut-être une lettre toutefois écrite dans un autre contexte. Ce document complémentaire offre un témoignage éclairant sur la manière dont le géomètre a pu envisager le principe de la formulation d'une langue universelle. Il s'agit d'une lettre datée de mars 1785, quelque huit ans plus tôt, adressée au comte Windischgraetz, l'un des acteurs de la réforme politique et administrative dans l'Empire autrichien. Condorcet y discute une initiative exceptionnelle et promise à l'échec : la création d'un concours organisé avec l'aide de plusieurs sociétés savantes européennes en vue de l'unification des actes juridiques et des contrats. Ce fut à notre connaissance, la première occasion pour Condorcet d'évoquer la formation d'une langue universelle.

> L'idée de ce programme [répond-il à son correspondant,] me paraissait digne de Leibniz ou de M. Turgot. […] Je conviens qu'il n'y a point peut-être d'impossibilité absolue de former cette expression générale d'un acte [juridique] quelconque que vous proposez, mais je craindrais qu'elle ne devienne si compliquée et qu'elle n'exigeât des expressions si éloignées de la langue commune qu'on ne finit par rejeter et tourner en ridicule une des idées les plus ingénieuses et les plus utiles qu'on ait jamais eues. […] Voici encore une autre difficulté. Cette formule pour être générale doit être rigoureusement

> traductible dans toutes les langues, à moins que vous ne vous contentiez d'une méthode générale de la former dont on serait tenu seulement de faire l'application à une langue ou à un pays. [16]

Cette lettre répond à beaucoup des questions que peut appeler le fragment écrit quelques années plus tard dans de tout autres conditions. Le géomètre suivait-il l'exemple de Leibniz ? Oui, mais de loin, comme celui d'un projet dont il avait eu connaissance des principes, mais dont il n'envisageait pas les détails[17]. Quel est son modèle ? Ici, comme souvent, il s'agit de Turgot, non pas seulement du ministre à sa maturité mais surtout du jeune métaphysicien qui avait contribué à divers articles inspirés par John Locke dans les premiers volumes l'*Encyclopédie* de Diderot et D'Alembert, tel l'article « Etymologie ». Après la mort de ce maître en 1781, Condorcet avait exposé au public sa philosophie en 1786 (*Vie de M. Turgot*) et sa politique en 1788 (*Essai sur les assemblées provinciales*). Ce même Turgot lui demandait, dans leurs correspondances précoces des premières années 1770, de rendre aussi explicites que possible les étapes mentales des démonstrations mathématiques même les plus simples.

> On exige seulement des démonstrations qu'elles soient rigoureuses, et comme il importe surtout d'aller en avant, on ne s'arrête pas à résoudre les difficultés métaphysiques qui se présentent, parce qu'on est sûr que l'habitude du calcul fera disparaître l'incertitude que ces difficultés semblent répandre. M. Turgot eût voulu qu'on dissipât jusqu'aux plus petites obscurités ; il eût voulu encore que l'analyste rendît compte des motifs qui lui font employer les opérations qui le conduisent à son but, qu'il montrât par quelles raisons il les a préférées, et par quelle suite de raisonnement elles se sont présentées à lui. […] On peut sans doute se dispenser de ces discussions si on ne regarde l'analyse que comme une science particulière, ou un instrument utile aux autres sciences ; mais elle cesse de l'être lorsqu'on la regarde comme une étude propre à former la raison, à la fortifier, et surtout à faire connaître la marche de l'esprit humain dans la recherche de la vérité. [18]

Telle était bien l'ambition de la langue universelle. Mais Condorcet avait encore un autre modèle. S'il ne le mentionne pas dans cette lettre de 1785, il l'indique en passant en traitant de la langue universelle de la géométrie. Il s'agit de Pierre Hérigone

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[16] Lettre de Condorcet à Windischgraetz, autographe de 4 p., datée du 26 mars 1785. Nous sommes redevable à Mme Martina Grečenková de nous en avoir procuré une copie. Sur cette correspondance et ce fonds, voir Martina Grečenková, « Les formules générales de tous les contrats imaginables… », *Studies on Voltaire and the Eighteenth Century*, 1 (2003), 271-289.

[17] Nous rejoignons ici les conclusions de Roselyne Rey formulées en 1994.

[18] Condorcet, *Vie de Monsieur Turgot* (Londres, 1786), 197-198.

dont le *Cursus mathematicus*[19], traité très employé dans l'enseignement de l'algèbre au XVII^e^ siècle[20], est à bien des égards énigmatique tant il est écrit dans une langue symbolique dont l'examen n'a pas encore fait l'objet d'une enquête complète. La consultation du *Cursus* permet de constater que Condorcet ne lui a pas seulement emprunté des éléments d'écriture hiéroglyphique de la géométrie élémentaire comme il l'a lui-même indiqué (774r/995). Non, c'est aussi la langue « méthodique » (tel est le vocabulaire de Condorcet pour désigner les relations entre les définitions, les hypothèses, les preuves et les conclusions) qui est marquée chez Hérigone par des signes particuliers, tout comme les relations d'ordre, et les « opérations générales de l'esprit ». Toutefois Condorcet ne paraît pas avoir eu sous les yeux l'ouvrage de son prédécesseur au moment où il a écrit et corrigé le *Fragment 4*. Il importe ici de rappeler que les fragments furent rédigés quand Condorcet vivait proscrit sous la Terreur révolutionnaire, caché à Paris, rue Servandoni. On ne sait pas de quels documents le savant a pu faire usage et tout porte à croire qu'il en a peu manipulé. Nous sommes donc conduits à conjecturer que quelque partie du *Cursus* était demeurée familière à l'esprit de notre académicien peut être depuis le temps de son apprentissage mathématique : en somme un modèle lointain intimement mûri qui a pu lui procurer un exemple de certitude dans les démonstrations.

Condorcet traite de plusieurs sciences mathématiques et physiques, et il a espéré faire de même pour les questions morales et politiques, programme qu'il s'était assigné dès le milieu des années 1780 (il l'a indiqué dans plusieurs ouvrages). Il a eu conscience du fait que les différents domaines ainsi passés en revue n'offraient pas les mêmes facilités. Chacun d'entre eux était à ses yeux comme un état particulier de la science qui dépendait à la fois de ses objets et d'un certain degré d'avancement des connaissances en la matière. Ici, comme déjà chez D'Alembert

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19 Pierre Hérigone, *Cursus mathematicus. Nova, brevi, et clara methodo demonstratus per notas reales et universales, citra usum cuiuscunque idiomatis intellectu faciles. Cours mathématique, démontré d'une nouvelle, brièfve, et claire méthode, par notes réelles et universelles, qui peuvent estre entendües facilement sans l'usage d'aucune langue* (Paris, 1634), t. 1 à 4 ; (Paris, 1637), t. 5.

20 Henri-Jean Martin, *Livre, pouvoirs et société à Paris, au XVII^e^ siècle, 1598-1701* (Genève, 1969). Giovanna Cifoletti, *La méthode de Fermat, son statut et sa diffusion : algèbre et comparaison de figures dans l'histoire de la méthode de Fermat* (Paris, 1990).

dans le *Discours préliminaire* de l'*Encyclopédie*, et comme plus tard chez Auguste Comte, le panorama des différentes sciences donne le spectacle de leur différentiation selon la capacité l'esprit humain à fixer et à explorer les objets que chacune se donne. De ce fait, la langue universelle de chaque science, bien qu'elle ait à obéir aux mêmes procédés que celles des autres, évoque *aujourd'hui* dans chaque occurrence une construction abstraite différente. C'est en effet tout d'abord une extension des signes de l'algèbre vers l'écriture d'opérations ou de distinctions jusque là laissées dans l'implicite[21]. Mais c'est aussi une exploration des formes élémentaires de la composition des textes mathématiques quand il est question des signes méthodiques. Plus loin c'est une manière de topologie qui s'exprime – on va le voir – dans celle de la géométrie ; une méthodologie de la mécanique analytique plus loin ; un protocole d'enregistrement des observations en astronomie ; enfin un programme que les successeurs immédiats de Condorcet appelleront « idéologie » à peine ébauché pour les sciences morales dans les derniers feuillets. Si bien que chaque science offre une classe particulière d'objets de dénombrements et de combinaisons, sans qu'il nous soit possible aujourd'hui de considérer que ces classes soient homogènes.

La langue de la géométrie de situation

Condorcet a traité sur plusieurs feuillets de la langue de la géométrie. Il lui a accordé une marque particulière : une barre oblique inversée ($\diagdown$). Il a ainsi noté $\diagdown\!\angle$ les angles, $\diagdown\!C$ les courbes, $\diagdown\!\llcorner$ les lignes et $\diagdown\!\llcorner\!\llcorner$ le périmètre d'une surface. Il a abordé à cette occasion la géométrie de situation « encore très peu connue, dont Leibniz et Euler ont donné quelques essais [et qui] a pour objet de déterminer non des grandeurs mais des relations de positions entre des points, des lignes, des plans, des solides » (767r/984). Avant d'écrire le mot « relations », Condorcet a tracé « *rapports* », puis il l'a biffé. Nous pouvons le comprendre comme une clarification visant à mettre tout à fait à l'écart les quantités et leurs rapports, c'est-à-dire les mesures.

Il a voulu établir une série de signes qui auraient désigné différentes positions dans l'espace, une fois considérés trois plans

[21] Seule cette langue universelle de l'algèbre étaient connue après la première édition de 1954.

qui se croiseraient au même point. Ces plans sont alors qualifiés d'horizontal, de méridien et de parallèle à la manière de l'astronomie (voir fig. 1 ci-dessous).

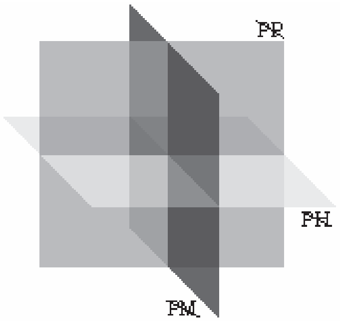


Figure 1. Système de trois plans arbitraires

Mais contrairement à nous, c'est *sans le secours de telles figures* que raisonne alors le géomètre[22] : la densité et la nature des biffures montre que, comme pour le reste du fragment sur la langue universelle, nous disposons de la feuille même sur laquelle s'est appuyée sa réflexion[23]. Chez Condorcet, point de tracé imagée de la pensée : les signes alphabétiques et hiéroglyphiques, écrits et combinés entre eux, suffisent à l'exprimer, à la mettre à l'épreuve, et à la transmettre.

Il esquisse des dénominations pour les huit régions de l'espace ainsi découpée, livrant la notation  pour la région zénithale, boréale et orientale (RZBE), puis son homologue  pour la région zénithale, boréale et occidentale (RZBO), et laissant deviner les six autres possibilités : deux du côté du zénith, l'australe et orientale (RZAE) et l'australe et occidentale (RZAO), et finalement quatre du côté du nadir, la boréale et orientale (RNBE), la boréale et occidentale (RNBO), l'australe et orientale (RNAE) et enfin l'australe et occidentale (RNAO) (voir fig. 2 ci-après).

[22] Gilles-Gaston Granger constatait déjà le peu de prédilection de Condorcet pour les figures, voir *La mathématique sociale du marquis de Condorcet* (Paris, 1956).

[23] On trouvera plus loin la reproduction de la seule figure tracée par Condorcet dans ce passage (768v/988).

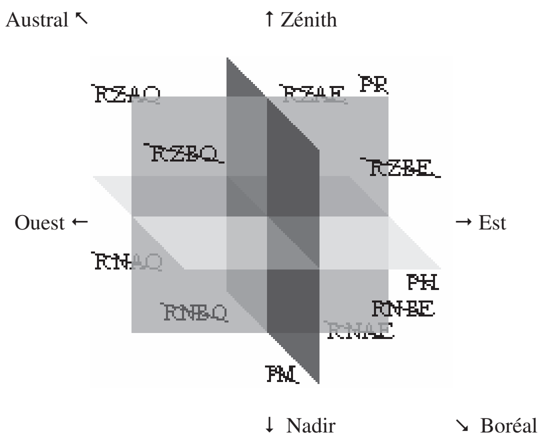


Figure 2. Définition de huit régions complémentaires de l'espace

De même la courte énumération « RBE, RBO, RE, RO » devait-elle suffire pour qu'un lecteur envisage d'une part trois cas induits par la position de deux plans et formés chacun de quatre régions complémentaires (voir l'exemple, fig. 3) et d'autre part les trois systèmes de demi-espaces possibles une fois un plan choisi (fig. 4).

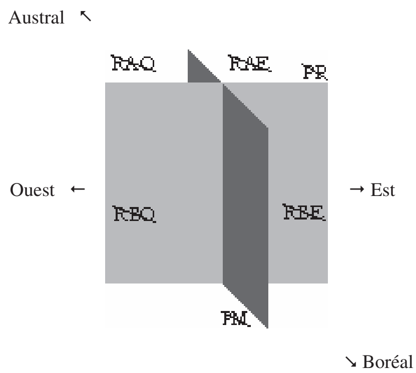


Figure 3. Quatre régions complémentaires
Ce sont les régions australe et occidentale (RAO), australe et orientale (RAE), boréale et occidentale (RBO), boréale et orientale (RBE). Même construction avec le plan horizontal (PH) et le méridien (PM), ou le plan horizontal (PH) et le parallèle (PP).

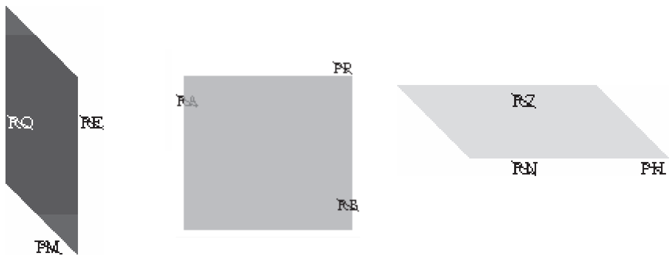


Figure 4. Trois systèmes de demi-espaces

Les régions ouest (RO) et est (RE) sont découpées par le plan méridien (PM) ; les australe (RA) et boréale (RB), par le plan parallèle (PP) ; celles du zénith (RZ) et du nadir (RN), par le plan horizontal (PH).

Il importe de remarquer que le géomètre n'a pas commencé par un découpage élémentaire en deux régions pour ensuite combiner ce premier acquis avec d'autres obtenus en introduisant des plans perpendiculaires. Il est parti, au contraire, de la donnée de tout l'espace, le découpant selon un référentiel de trois plans. Ensuite, par une opération mentale consistant à éliminer d'abord un, puis deux plans, il a déduit de sa construction des régions qui auraient pu nous paraître évidentes au premier abord. Ce cheminement manifeste que les données (*data*) les plus simples ont été, pour Condorcet, l'unité de l'espace et un principe général de découpage. Tout se déduit ici de la décomposition d'une unité initiale[24]. Le géomètre ne raisonnait pas autrement quand il envisageait des questions morales et politiques : en premier lieu vient l'humanité, en second un principe de division dont l'arbitraire doit être discuté : l'abus d'autorité intellectuelle que présuppose la formation de castes de prêtres, le peu de raison d'une différence des droits entre les hommes et les femmes, ou encore les injustifiables privilèges des blancs au détriment des noirs. Autant de thèmes qu'illustrent amplement l'*Esquisse* ou diverses positions prises par Condorcet pendant les dernières années de l'Ancien régime et la Révolution. Ici le geste de tradition savante, longuement mûri chez le géométrie, a coïncidé

[24] Pour d'autres manifestations de cette prédilection pour la décomposition de l'unité chez Condorcet et chez ses confrères géomètres, voir *La Mesure d'Etat*, cité plus haut.

avec le mouvement d'opinion politique qui s'est exprimé dans le mot d'ordre républicain de 1792 : « Unité et indivisibilité de la République ».

Le fait que les trois plans initiaux sont arbitraires, que la notation demeure relative à ce premier référentiel, et qu'il serait aisé d'en changer est traité par Condorcet sur le ton de l'évidence. Il suffirait pour les notations de la géométrie comme pour celles de l'algèbre d'introduire les autant d'esprits (prime, seconde, *etc.*) que cela serait nécessaire. En effet, l'écriture des objets est relative aux hypothèses.

> On sent que par ce moyen, on peut désigner toutes les conditions relatives à l'hypothèse de considérer les points, les lignes, les plans, les surfaces comme ayant des faces ou des côtés. En effet, si après avoir fait la supposition de trois plans déterminés, l'un comme horizontal, l'autre comme méridien, l'autre comme parallèle, on veut désigner les régions par rapport à d'autres plans, rien ne sera plus simple en appelant P'H', et ainsi de suite, le nouveau plan qu'on appelle horizontal » (768r-768v/988 ; nous illustrons ce passage au moyen de fig. 5).

Quelques lignes d'exemple

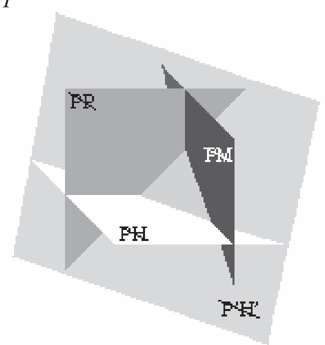


Figure 5. Recours à un nouveau plan

Il s'est agi ensuite de faire entendre comment employer la notation qui venait d'être proposée. Cela tient en une dizaine de lignes.

Pour la transcription nous avons accentué le tracé du fil CDE à proximité de D. Dans l'édition de 2004, nous avons exploré la combinaison des signes et le schéma en considérant que, par rapport au plan de la feuille de papier, CD se trouverait en arrière et DE en avant. Nous obtenions trois reconstitutions, certes

768v/988

Soit le fil AB sur un

plan que le fil CDE

passe au-dessous du fil AB

en D tourne autour et repassant

par dessus aille vers DE.

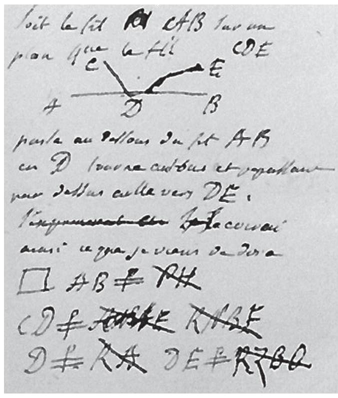
J'écrirai

ainsi ce que je viens de dire

□ *AB* ⋕ PH

CD ⋕ RNBE

D ⋕ RA *DE* ⋕ RZBQ



compatibles mais toutes imparfaites (p. 988-989). Pour le présent article nous considèrerons maintenant que CD se trouve en avant et DE en arrière. Les formules se révèlent dans ces conditions adéquates. Scruter l'original, la photographie et ses dérivés, songer au mouvement de la main de Condorcet, s'interroger sur l'encrage de la plume au moment où CDE arrive en D, imaginer une attention particulière de Condorcet au moment de tracer le passage en D : autant de spéculations impossibles à trancher. Un seul fait est susceptible d'être établi. Le manuscrit des formules, la paraphrase que nous établissons et le dessin que nous proposons aujourd'hui sont conformes entre eux. Ils n'appellent pas de correction des traces laissées par Condorcet[25].

On peut suivre sur la fig. 6, ci-dessous, cette lecture conditionnée par une interprétation du dessin (selon laquelle CD se trouverait en avant et DE en arrière).

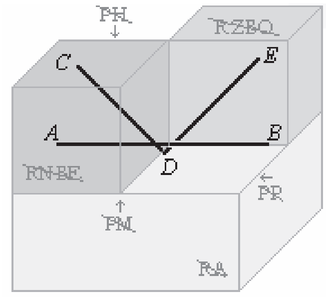


Figure 6. Interprétation du passage sur le croisement des fils AB et CDE

L'unique exemple de la géométrie de situation offert par Condorcet est donc pertinent : la combinaison des notations proposées permet de suivre rigoureusement le détour d'un fil autour d'un autre.

Le génie mathématique

A quoi pouvait donc songer notre savant en donnant cet exemple de géométrie de situation ? Une douzaine d'années plus tôt, dans ses fonctions de Secrétaire de l'Académie royale des sciences de Paris, il avait livré au public une réflexion éclairante à propos de la géométrie de Leibniz. L'occasion lui en fut donnée par l'éloge

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[25] La reconstitution proposée en 2004 conduisait à :

	□ AB ╪ PH	CD ╪ [RZAQ]	D ╪ [RN]	DE ╪ [RZBE]
ou bien à	□ AB ╪ [PR]	CD ╪ RN-BE	D ╪ RA	DE ╪ RZBQ
ou bien à	□ AB [⊥] PH	CD ╪ RN-BE	D ╪ RA	DE ╪ RZBQ

en choisissant ⊥ pour désigner la relation de perpendicularité que Condorcet avait mentionnée quelques feuillets plus tôt, mais sans en fixer le signe : « Il sera nécessaire dans l'usage d'avoir des signes pour désigner le parallélisme, l'intersection perpendiculaire, les degrés d'un angle, les diverses figures, les tangentes, les normales et les courbes dont on donne une théorie plus détaillée. » (765r/982).

qu'il avait à rendre à Jacques de Vaucanson (1709-1782) peu après sa mort. Condorcet admirait profondément le mécanicien[26].

> Le génie dans cette partie des sciences [la mécanique], consiste principalement à imaginer et à disposer dans l'espace les différents mécanismes qui doivent produire un effet donné, et qui servent à régler, à distribuer, à diriger la force motrice. Il ne faut point regarder un mécanicien comme un artiste qui doit à la pratique ses talents ou ses succès. On peut inventer des chef-d'œuvres en Mécanique sans avoir fait exécuter ou agir une seule machine, comme on peut trouver des méthodes de calculer les mouvements d'un astre qu'on n'a jamais vu. Dans la plupart des autres parties des sciences, on trouve des principes constants, une foule de méthodes offrent au génie une source inépuisable de moyens. Si un savant se propose une question nouvelle, il l'attaque avec les forces réunies de tous ceux qui l'ont précédé. Il n'en est pas ainsi de la Mécanique, sa véritable théorie dépend de cette Géométrie de situation dont Leibniz a connu l'existence, mais qui n'a fait encore que peu de progrès » (HARS 1782, p. 160-161).

L'exemple donné par Condorcet dans sa langue universelle illustrait donc à ses yeux ce rapport de la géométrie de Leibniz à la mécanique des machines de Vaucanson. Le géomètre nous confirme au passage le mode opératoire qui perçait sous le manuscrit de la langue universelle : il s'agit de se donner les moyens de « disposer dans l'espace » les éléments dont les relations doivent être combinées en vue d'un effet recherché.

C'est dans une autre section du fragment sur la langue universelle que Condorcet nous procure sa définition de la « mécanique des machines » (771v-773r/993-994) par opposition à la « mécanique rationnelle » (770r-771v/990-993)

> [La mécanique des machines] renferme deux objets : 1° le calcul des forces rigoureux ou réduit aux résultats pratiques ; 2° la disposition des parties de chaque machine, et la manière dont elles produisent leur action. [… Il] est aisé de voir que [le premier] objet peut être facilement rempli à l'aide de planches et de la langue déjà établie. Il en sera de même de la seconde partie. Mais on verra que pour l'une et pour l'autre, on sera obligé d'augmenter le vocabulaire d'un assez grand nombre de signes ; mais que tous pourront s'expliquer à l'aide de la langue même. On ne pourrait former ce vocabulaire pour la seconde partie qu'après avoir composé un traité analytique de cette partie de la mécanique sur laquelle il n'existe encore aucune théorie.

Ainsi Concorcet envisageait-il la possibilité d'une langue propre à la mécanique des machines, celle de son vocabulaire, de ses

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[26] Outre l'obligation académique de cet éloge qui fut imprimé dans l'*Histoire de l'Académie royale des sciences pour l'année 1782* (Paris, 1785), 156-168, Condorcet n'a pas hésité à hisser Vaucanson au rang du physicien Newton et du chimiste suédois Bergman, parmi les esprits les plus élevés que les « sociétés civilisées » aient pu produire (*Tableau historique*, « Fragment 1 », 547^{bis}r/513).

planches et de ses signes, alors même qu'il admettait que cette théorie en était aux prémisses, seulement esquissée dans la géométrie de situation de Leibniz, et que l'on ne connaissait presque rien du rapport de cette géométrie-ci à cette mécanique-là. A quoi tenait une telle spéculation ? Seulement au fait que la mécanique des machines était susceptible d'une théorie analytique et que la géométrie de situation avait pour objet non les grandeurs mais les relations entre les positions des choses. Dès lors l'exposition et la construction d'une langue universelle de la mécanique des machines étaient concevables.

Un nouvel élément imputable aux préparatifs du *Tableau historique*, l'« Essai sur la faculté appelée génie » (578r-609v/731-760), nous éclaire sur l'exercice de la langue universelle. Il relève de l'art de combiner les moyens de manière abstraite, et cet art dans les mathématiques comme dans la conception des machines est tout entier d'écriture, de désignation, de disposition et combinaison.

> Du moment où l'homme a pris l'habitude d'attacher constamment des signes à ses idées, l'imagination peut s'exercer sur ces signes eux mêmes. Ainsi par exemple les idées abstraites que la géométrie considère sont exprimées par des lignes, et le géomètre qui veut suivre de tête une démonstration ou qui cherche à résoudre un problème imagine des combinaisons ou lignes, et se représente des figures comme si elles étaient déjà tracées sur un tableau. L'algébriste est également obligé d'imaginer des formules, des opérations, s'il veut ou calculer sans écrire, ou trouver une vérité nouvelle. Ni l'un ni l'autre ne pourraient rien exécuter au delà de ce que leur mémoire leur rappelle s'ils ne se représentaient d'avance dans leur pensée, s'ils n'imaginaient les figures, les formules, les opérations qui sont les instruments de leur science comme un mécanicien, un physicien, un chimiste imaginent des instruments, des appareils ou des machines. Ainsi puisque ces images soit d'objets soit de signes sont comprises sous le nom général d'idées, puisque leurs combinaisons se forment de la même manière, dans une même intelligence et sont soumises aux mêmes lois, puisque ces combinaisons d'images font partie des opérations nécessaires pour trouver la vérité même dans les sciences les plus abstraites et qui paraissent au premier coup d'oeil le partage exclusif de l'entendement pur, toute distinction eût été plus nuisible qu'utile à la recherche de ce qui caractérise véritablement le génie dans les différentes classes des Sciences ou des Arts. (586r-v/737)

Point de figure, ou presque on l'a vu, sur la feuille même, mais non pas point d'image à l'esprit. Les signes de la langue universelle peuvent bien désigner des quantités, des classes de quantités, des choses concrètes, des espèces ou des figures. Ils offrent au géomètre un point d'appui mnémotechnique mais abstrait, et cette abstraction étant posée ils obéissent – à ses yeux – aux même lois de composition. La langue universelle est, au sens strictement étymologique du mot, une idéo-logie combinatoire.

> Dans la mécanique pratique, dans les arts qui en dépendent, les véritables découvertes supposent nécessairement la combinaison nouvelle des moyens simples de modifier et de ménager les forces, de diriger, de régler les mouvements, de disposer les diverses parties d'une machine, de varier ces dispositions afin de produire un effet donné, et comme cette combinaison nouvelle doit se former dans la pensée, du moins par partie avant d'être exécutée, il en résulte évidemment que toute invention en ce genre se réduit encore à une nouvelle combinaison d'idées. (591v/742)

Du croisement des fils aux noeuds

Il n'est pas interdit de s'y essayer : Condorcet n'a-t-il pas voulu procurer à ses lecteurs éventuels les balbutiements de la langue plutôt que la sceller (750r/957) ? Ainsi par exemple le géomètre a proposé de définir les lignes *qui n'étaient pas dans un seul plan*, les lignes à double courbure au moyen des signes ⊾⊃⊂ (766r/983). Or, le contournement du fil AB par le fil CDE n'était qu'un exemple élémentaire, à l'instar de l'équation du cercle ou de celle de l'ellipse toutes deux données ailleurs (764v/981). Combinons ces éléments.

La figure 6 pourrait en effet être accompagnée de diverses formules de définition du type □ ⊾⊃⊂ ...ABCDE... : « soit une ligne non plane d'abord indéfinie, puis passant par les points A, B, C, D et E, puis à nouveau indéfinie ».

⊾⊃⊂ ...CDEBA... désigne une boucle croisée au point D, pour autant que la ligne EB demeure dans la région zénithale, boréale et occidentale (EB ⋕ RZBQ, voir la fig. 7). ⊾⊃⊂ ...BACDE... parcourrait une boucle simplement croisée analogue à la précédente dès que la ligne AC obéirait à une propriété homologue (AC ⋕ RNBE). ⊾⊃⊂ ...ABCDE... serait encore une boucle croisée, mais cette fois au prix d'une condition un peu différente : BC devrait demeurer dans la région boréale du nadir (BC ⋕ RBN). Enfin ⊾⊃⊂ ...BAEDC... serait une quatrième boucle croisée si la ligne AE demeurait dans la région zénithale et boréale (AE ⋕ RBZ).

⊾Mais dans les deux derniers cas, rien n'empêche de suivre un autre chemin : entre B et C par exemple pour le premier d'entre eux (⊾⊃⊂ ...ABCDE...). En ajoutant un point F dans la région zénithale, boréale et orientale (à l'arrière-plan et à gauche sur la fig. 7), en s'assurant que BF – tout en restant dans la région zénithale – ne traverse pas celle où se trouve E, puis que FC demeure dans le quart d'espace boréal et oriental (le chemin FC serait direct), alors ⊾⊃⊂ ...ABFCDE... ne décrirait plus une boucle croisée, mais un noeud. Ce chemin, moins simple, est suivi au moyen des formules indiquées ci-dessous[27].

□ LX …ABFCDE…

AB ǂ PH

F ǂ RZBE

BF ǂ RZ

BF‡ǂ RZBQ

FC ǂ RBE

CD ǂ RN-BE

D ǂ RA

DE ǂ RZBQ

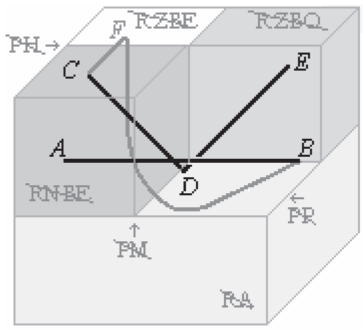

Figure 7. Bouclage d'un noeud

Est-ce à dire que les quelques lignes de la langue universelle où Condorcet se propose de donner les bases de la géométrie de situation constitueraient un prototype de la théorie des nœuds que les spécialistes de la topologie connaissent aujourd'hui ? Une telle phrase n'aurait pas grand sens tant il est certain que ce passage du *Fragment 4* n'a connu aucune publication jusqu'en 2004, et que sans doute il n'a eu aucune postérité. Mais il est clair toutefois que cette occurrence de la géométrie de situation, en abordant la question de l'analyse combinatoire des croisements de fils, s'est approchée au plus près des distinctions que la topologie depuis la seconde moitié du XIXe siècle rapportera à une théorie bien différente de celle à laquelle paraît avoir songé Condorcet à propos de la mécanique des machines[28].

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27 Nous avons utilisé le signe d'impossibilité ‡ qui est défini par Condorcet en 756r/965.

28 Dans l'édition de 2004, p. 989, nous avions donné un exemple plus abstrait dans le même sens. En effet, un fil croisé pourrait s'écrire ainsi :

□ LX ...ABCDEFG... □ ...AB ǂ RZAE □ BC ǂ RZBE

Mémoire et perfectionnement des sciences

Construire une langue qui pourrait rende compte du mouvement d'une navette mécanique et par conséquent de la combinaison de moyens engagés dans la construction d'un métier à tisser, tels ceux auxquels le nom de Vaucanson est associé, fut pour Condorcet l'occasion d'indiquer de manière étonnamment explicite comment un géomètre de son temps s'essayait à concevoir un objet ou une question des plus abstraites dans le corpus de la géométrie de la fin du XVIII^{e} siècle. A ce titre les quatre pages sur la langue de la géométrie de situation livrent une clé de l'entreprise du *Fragment 4*. La langue universelle n'est aucun des développements anachroniques que sa grande abstraction pourrait suggérer au premier regard, une fois passés deux siècles d'autres sciences.

Plusieurs fois au cours du fragment, Condorcet a tenté de récapituler les propriétés qu'il attribue à la langue nouvelle. A ses yeux elle n'était pas en premier lieu une norme mais une marque d'une manière habile de combiner les moyens que procurerait au savant l'expérience de ses prédécesseurs et de ses contemporains. Il ne faut donc pas y chercher une vaste architecture épistémologique, telles celles proposés aux cours des siècles suivants, mais y reconnaître la mise en œuvre artisanale d'une sorte de rationalisme appliqué, forgé dans l'expérience scientifique de la seconde moitié du XVIII^{e} siècle, inscrit dans la tradition algébrique inaugurée au XVI^{e} siècle et gouverné par une hypothèse forte sur la langue des sciences et par conséquent sur l'écriture, sur l'imprimerie et sur la société des savants.

Cette hypothèse – longuement discutée dans les matériaux qui devaient former le *Tableau historique* et abrégée sans nuance

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□ CD ≠ RZBQ □ DE ≠ RNAQ □ EF ≠ RNAE □ FG… ≠ RNBE

et un fil noué :

□ [illegible] …ABCDEFG… □ …AB ≠ RZAE □ BC ≠ RNBE

□ CD ≠ RZBQ □ DE ≠ RZAQ □ EF ≠ RNAE □ FG… ≠ RZBE

L'écriture transpose alors le parcours des deux fils en une combinatoire. La notation met en évidence les nombres de passages des fils au travers des plans PH (une fois pour le fil croisé contre quatre fois pour le fil noué), PM et PR (identiques dans les deux cas). Cette comparaison suggère l'irréductibilité de la forme du fil noué à la forme du fil croisé.

dans les colonnes de l'*Esquisse* en une évocation répétitive des bienfaits de l'imprimerie – consiste à considérer que les objets de science ne sont accessibles à l'entendement humain que par le filtre d'une langue proprement savante affranchie des langues communes mais toujours susceptible d'être exprimée par elles. Ce filtre est par voie de conséquence aussi celui de l'écriture de cette langue proprement scientifique, et celui des moyens matériels de sa transmission – en particulier, nous y arrivons enfin, celui de l'imprimerie à l'époque moderne. Cette langue étant admise et établie sous une forme donnée et adéquate à telle science, les objets de cette science sont supposés ou bien déterminées ou bien indéterminés. Dès lors la langue universelle vise à exprimer sans ambiguïté la détermination de ces objets ou bien le degré de leur indétermination, cela sans le secours des langues communes. Trois principes de perfectionnement en découlent que Condorcet a exploré tout au long du *Fragment 4*: établir des définitions, multiplier les combinaisons, qualifier le degré de certitude à espérer. Ainsi l'art de raisonner du géomètre consistait-il à disposer les signes, puis à les combiner en un tableau des possibles ou une réduction au moyen du calcul.

Cet art, s'il s'exerçait sur les traces matérielles de l'écriture, n'en était pas moins tout intérieur. Or comment à l'époque de Condorcet réglait-on le cheminement intérieur d'une pensée ? En la soumettant aux normes établies de l'expression orale. Si, dans sa langue universelle, les notations sont particulièrement abstraites, elles n'en demeurent pas moins, par principe et dans l'usage attendu, conformes à cette nécessité que tout lecteur doit pouvoir les verbaliser immédiatement dans sa langue vernaculaire[29]. Conçue pour faire entendre les choses connues et administrer des preuves, elle relève donc nécessairement d'une rhétorique qui doit opérer dès qu'on verbalise un énoncé, cela dans toutes les langues possibles. Mais, dès lors, les traces écrites doivent obéir à une double nécessité : d'une part abstraire l'écriture de toute référence à quelque langue vernaculaire que

[29] L'utopie de la langue universelle condorcétienne n'est pas véhiculaire : elle est au contraire supposée commune à toutes les langues vernaculaires et, qui plus est, au latin des savants. Nous n'employons pas le terme de langue « naturelle » comme on serait tenté de le faire aujourd'hui, car pour notre géomètre, en cela parfaite incarnation des Lumières, cette langue universelle est plus naturelle que tout autre.

ce soit et d'autre part épouser toutes ces langues pour que les formules soient dites et pensées par qui que ce soit. La syntaxe des opérations élémentaires de combinaison et de disposition satisfait une telle condition. De là l'engendrement d'un projet de langue ébauché dans ce manuscrit mais déjà dotée, on l'a vu, d'un fort potentiel heuristique. Condorcet entendait livrer aussi explicitement que possible à la fois l'exemple et le principe d'une telle langue. Il nous a procuré, ce faisant, un témoignage rare de l'art de la géométrie à la fin du XVIII[e] siècle.